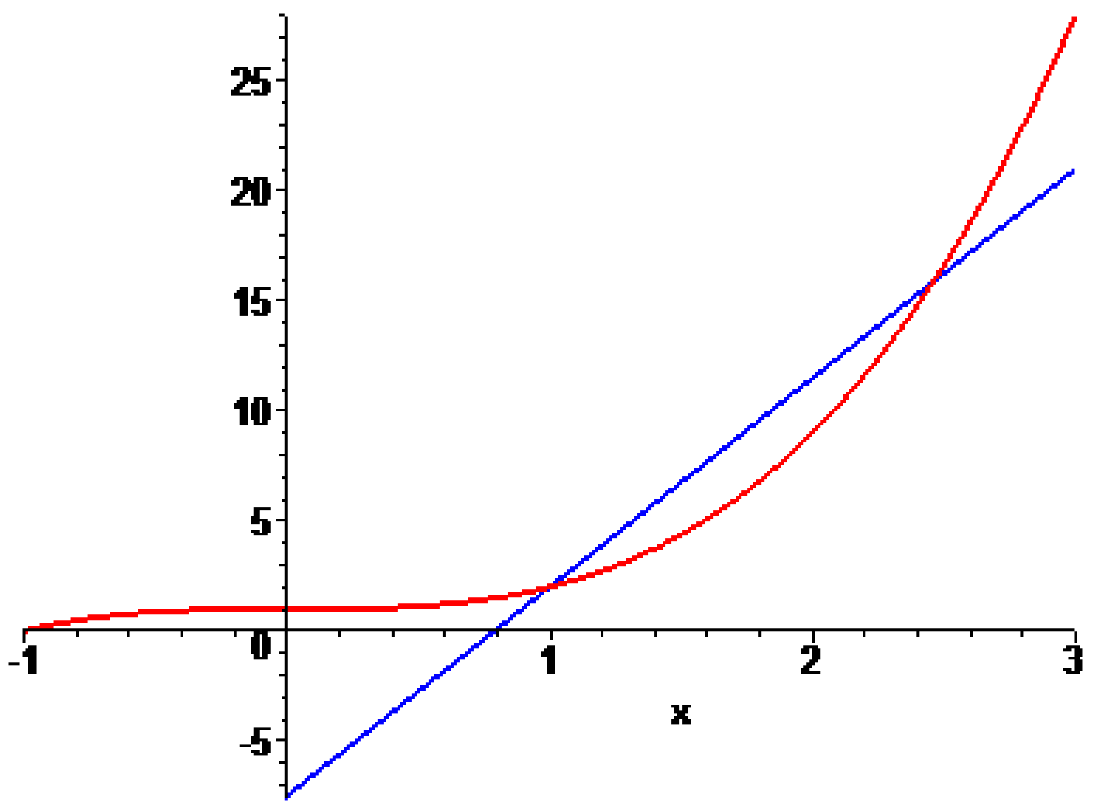
LESSON 1.4

THE TANGENT AND VELOCITY PROBLEMS

SECANT LINES APPROACHING THE TANGENT LINE



Time interval	[2,2.5]	[2.25,2.5]	[2.4,2.5]	[2.49,2.5]	[2.4999,2.5]
Average velocity					

Consider dropping a ball from the top of a 100 foot parking deck. The height $h(t)$ of the ball t seconds after it is dropped is modeled by $h(t) = 100 - 16t^2$. It can be shown that the ball hits the ground after $t = 2.5$ seconds (How?). The average speed of the ball over any time period is given by $r = \frac{\Delta h}{\Delta t} = \frac{\text{change in height}}{\text{change in time}}$. If we wished to calculate, on average, how fast the ball was moving in its last half-second, this would be

$$r = \frac{\Delta h}{\Delta t} = \frac{\text{change in height}}{\text{change in time}} = \frac{h(2.5) - h(2)}{2.5 - 2} = -72 \text{ feet/sec.}$$

Complete the table above and answer the question:

How fast is the ball traveling at the moment of impact?

PROBLEM

Consider the quantity $\boxed{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots}$

Let the function $f\left(n\right)$ stand for "the sum of the first n successive terms in the above expression, starting with $\frac{1}{2}$." For example, $f\left(3\right)=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=0.875$. Complete the following table (and simplify your results). Can you write this using limit notation?

n	**1**	**2**	**3**	**4**	**5**
$f\left(n\right)$					

PROBLEM

Drop a ball from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Galileo once said, "The distance traveled is proportional to the square of the time it has been falling." In modern terms, we know $D(t) = 4.9t^2$. Find a way to determine the velocity of the ball after 3 seconds. Explain your answer.