MTH 150 **Name:**
Exam 3
Spring 2014

DIRECTIONS: This is a <u>closed book</u>, <u>closed notes</u> exam. <u>No electronic devices</u> are allowed (this means calculators, computers, cell phones, pagers, etc.). Be neat and <u>show all work</u> (except where indicated) to receive full credit. Correct answers without the supporting evidence to back it up receive only partial credit. Good luck.

Problems 1-6: Answer **TRUE** or **FALSE**. No work is necessary. **(1 point each)**

1. True The maximum of a function that is continuous on a closed interval can occur at two different values in the interval.

2. False The product of two increasing functions is increasing.

3. True ! The graph of $y = \cos x$ has an infinite number of inflection points.

✓ 4. False If $y = f(x)$, f is increasing and differentiable, and $\Delta x > 0$, then $\Delta y > dy$.

5. False The Mean Value Theorem can be applied to $f(x) = \frac{1}{x}$ on the interval $[-1, 1]$.

6. True Rolle's theorem is an illustration of the Mean Value Theorem.

Problems 7-9: Circle the letter that corresponds to your answer. **(5 points each)**

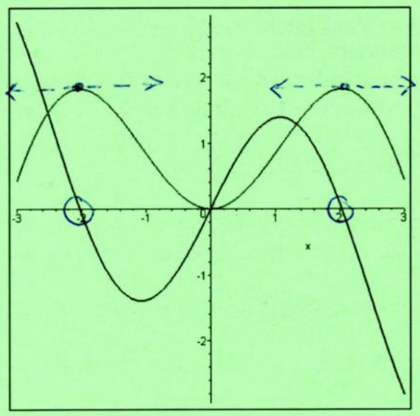
7. If both $f'(x) < 0$ and $f''(x) > 0$ on an interval, then the graph of $f(x)$ is
   - ~~A.~~ increasing/concave upward
   - ~~B.~~ increasing/concave downward
   - C. decreasing/concave upward
   - D. decreasing/concave downward
   - ~~E.~~ None of these

✓ 8. The Second Derivative Test is used to…
   - A. determine intervals of increase/decrease
   - ~~B.~~ determine the concavity of the graph
   - C. find relative extrema
   - D. find points of inflection
   - E. None of these

9. Consider the function $f(x) = e^{-x^2/2}$. Determine where $f(x)$ is concave upward.
   - A. $(-\infty,-1)\cup(1,\infty)$
   - B. $(-\infty,0)\cup(0,\infty)$
   - C. $(1,\infty)$
   - D. $(-1,1)$
   - E. $(0,\infty)$

-0

10. **(8 points)** Below are the graphs of a function f and its derivative f', both in the same viewing window. Which is which? Write a short summary and support your claims with detailed reasoning.

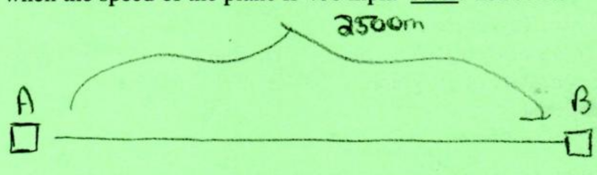


SUMMARY:

The graph that is lighter and does Not Dip beneath the x-axis is the original function while the other is its derivative. This can be seen ~~to~~ when the original function peaks and has a Horizontal Tangent value of zero, the graph of the derivative reaches the x-axis at that point ='ing zero.

Good

11. **(8 points)** A plane begins its takeoff at 2:00 P.M. on a 2500 mile flight. After 5.5 hours, the plane arrives at its destination. Explain why there are at least two times during the flight when the speed of the plane is 400 mph. Note: $2500 / 5.5 \approx 455$



$f'(c) = \frac{f(b)-f(a)}{b-a}$

Average Speed = 455 mph

$D = RT$

$R = \frac{D}{T}$

$R = 455$ mph

The average Speed of the plane was 455 mph, but the Plane had to exceed this speed to account for take-off/land speeds. Using the Reasoning ~~by~~ behind Rolle's thm, Twice we can say the plane had to reach a speed of 400mph; once while accelerating to reach peak speed (> 455) and once while Deccelerating to reach final speed (0).

Great!

-0

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DIRECTIONS: Calculators are permitted on this part of the exam. However, answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

12. **(10 points)** Locate the absolute extrema of the function $y = 3x^{2/3} - 2x$ on the interval $[-1, 1]$. *check endpts

$y' = \frac{2}{3}(3x)^{-1/3} - 2$

$y' = 2x^{-1/3} - 2$

$y' = \frac{2}{x^{1/3}} - 2$

set = 0

x can't be 0

$2x^{-1/3} - 2 = 0$

$2x^{-1/3} = 2$

$x^{-1/3} = 1$

$x = 1$

x	y
-1	5
1	1
0	0

abs. max: (-1, 5)
abs. min: (0, 0)

13. **(10 points)** The measurement of the radius of the end of a log is found to be 14 inches, with a possible error of $\frac{1}{4}$ inch. Use differentials to approximate the possible propagated error in computing the area of the end of the log.

$r = 14$ error/dr $= 1/4$

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$dA = 2\pi r\,dr$

$= 2\pi(14)(\frac{1}{4})$

$dA = 21.99114858\ in^2$

The possible propogated error for this problem is $21.99\ in^2$

in a percent, thats $\frac{21.99}{(14)^2\pi} \times 100$

3.57% error.

Good

-0

14. Consider the function $y = x^3 - 3x^2 + 3$.

(a) **(2 points)** Find all of the critical numbers of f.

$y' = 3x^2 - 6x = 3x(x-2) = 0$

$3x = 0$, $\boxed{x=0}$ ✓ $x - 2 = 0$, $\boxed{x=2}$

CR#

(b) **(3 points)** Find the intervals on which f is increasing and decreasing.

Rel max Rel min

(+) (−) (+)

0 2

inc: $(-\infty, 0) \cup (2, \infty)$ ✓

DEC: $(0, 2)$ ✓

(c) **(2 points)** Locate any relative extrema.

Rel max: $(0,3)$ ✓ Rel min: $(2,-1)$ ✓

(d) **(3 points)** Find the intervals on which f is concave upward and concave downward.

$y'' = 6x - 6$ → $6x - 6 = 0$, $\frac{6x}{6} = \frac{6}{6}$, $\boxed{x = 1}$

CC↑: $(1, \infty)$ ✓

CC↓: $(-\infty, 1)$ ✓

(−) (+)

1

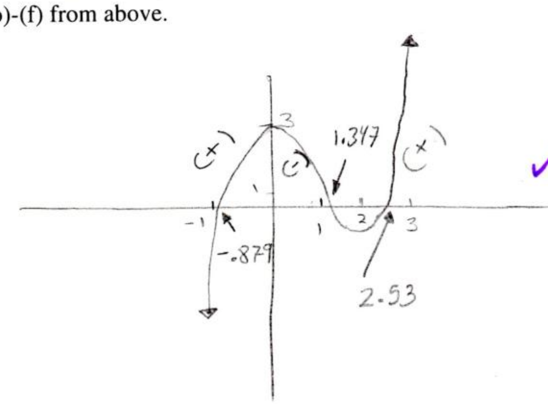
(e) **(2 points)** Find any points of inflection.

POI: $(1,1)$ ✓

(f) **(2 points)** Estimate any x-intercept(s). Use your calculator.

x-intercept: $-.879$, 2.53, 1.347 ✓

(g) **(5 points)** Make a complete, clear, neat sketch of the graph of f by using parts (b)-(f) from above.



✓

-0

15. **(12 points)** Choose <u>**ONE**</u> of the problems below. Place a checkmark next to the problem you are doing.

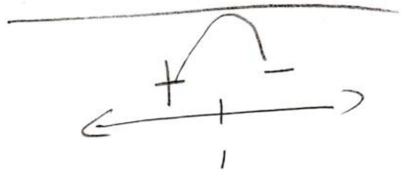
✓ <u>Problem A</u>: Find the critical number(s) of the function $f(x) = xe^{-x}$, if any. Additionally, find the open intervals on which the function is increasing or decreasing and locate all relative extrema.

______ <u>Problem B</u>: Find all points of inflection to the graph of $y = \sin x$ and discuss the concavity of the graph of the function.

$x \cdot -1e^{-x} + 1 \cdot e^{-x}$

$-xe^{-x} + e^{-x}$

$e^{-x}(-x+1)$ ✓



$(1)(e^{-1})$

Critical #: 1

✓ Increasing: $(-\infty, 1)$

Decreasing: $(1, \infty)$

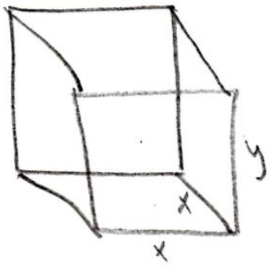
✓ Extreme max: $(1, e^{-1})$

−0

−1 //

16. **(12 points)** An airline policy states that all box-shaped luggage may not have the sum of length, width, and height exceed 64 inches. What are the dimensions of a square-based box that has the greatest volume under these conditions? **Be sure to show all work here.**

$3x = 64$



✓ $2x + y = 64$ (secondary equation)

$(x)(x)(y) = \text{volume}$

✓ $x^2 \cdot y = v$ (primary equation)

$x^2 \cdot (64 - 2x) = v$

$v = 64x^2 - 2x^3$ ✓

$v' = 128x - 6x^2$

$0 = 2x(64 - 3x)$

$x > 0$

$x < 32$

$2x = 0$, $x = 0$

$64 - 3x = 0$

$-3x = -64$

$\boxed{x = 21.33}$ ✓

maximum.

$128(20) - 6(20)^2 = 160$

$128(22) - 6(22)^2 = -88$

$+$ 0 ⟶ 21.33 ⟶ $-$ sign of v' ✓

$2(21.33) + y = 64$

$y = 21.34$

$$\boxed{\begin{aligned} x &= 21.33 \text{ inches} \\ y &= 21.34 \text{ inches} \end{aligned}}$$

Good

$x = y$

−0