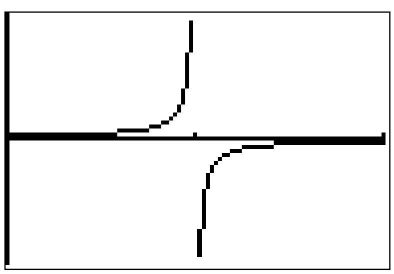
MATH 166
Lesson 1.5b
Limit of a Function (Infinite Limits)

Recall from earlier discussions that writing $\lim_{x \to c} f(x) = L$ tacitly assumes that *L* is a finite number. In this section, we will study statements such as $\lim_{x \to c} f(x) = \infty$. Since this literally says that $f(x)$ grows without bound as *x* approaches *c*, we will often write $\lim_{x \to c} f(x)$ DNE in place of $\lim_{x \to c} f(x) = \infty$. The statement $\lim_{x \to c} f(x) = \infty$ doesn't support the idea of $f(x)$ approaching any one number so saying the limit fails to exist is justified. Here is an example that illustrates some infinite limits.

<u>Example</u>: Investigate the limiting behavior of the following functions for values *not* in the domain: (a) $h(x) = \frac{1}{2-x}$ and (b) $g(x) = \frac{1}{(2-x)^2}$.

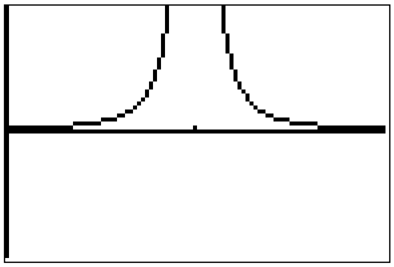
<u>Solution</u>: Let's take a look at each on the calculator.

(a) First, it is clear that the domain of $h(x)$ is all real numbers with the exception of $x = 2$. If you plot $h(x) = \frac{1}{2-x}$ in the window $[1, 3, -50, 50]$, this is evident:



We can see from the picture that $\lim_{x \to 2^+} h(x) = -\infty$ and $\lim_{x \to 2^-} h(x) = \infty$. It is definitely the case that $\lim_{x \to 2} h(x)$ DNE. However, notice that each of the statements $\lim_{x \to 2^+} h(x) = -\infty$ and $\lim_{x \to 2^-} h(x) = \infty$ indicate that the one-sided limits fail to exist also (since $\pm\infty$ are not real numbers).

(b) In a similar way, you will notice the same restriction on the domain for $g(x)$. Here is the graph (using the same window settings):



In this case, $\lim_{x\to 2^+} g(x) = \infty = \lim_{x\to 2^-} g(x)$ so we could write $\lim_{x\to 2} g(x) = \infty$. However, writing $\lim_{x\to 2} g(x)$ DNE is also fine since ∞ is not a number. You should notice that both of the graphs have a **vertical asymptote** at $x = 2$. Here is the definition.

Definition: If $\lim_{x\to c} f(x) = \pm\infty$ from the left or the right, then the vertical line $x = c$ is a **vertical asymptote** to the graph of $y = f(x)$.