MATH 166
Lesson 2.1
Derivatives & Rates of Change

You are strongly encouraged to look back at Lesson 1.4; this lesson covers the same themes but formalizes the ideas. There, we looked at the following diagrams

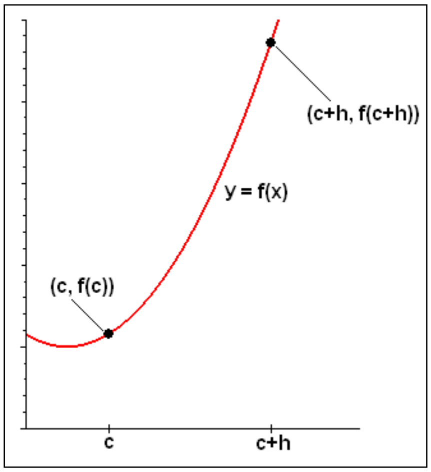


Figure 1.

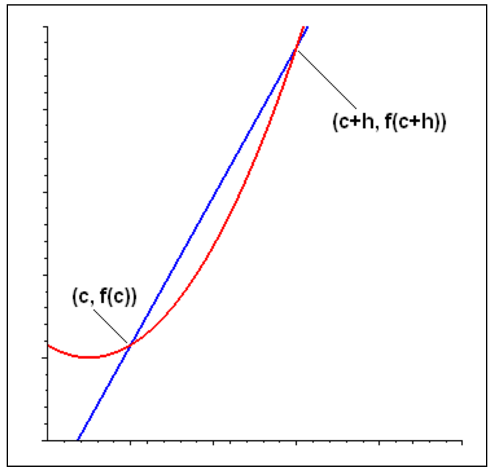
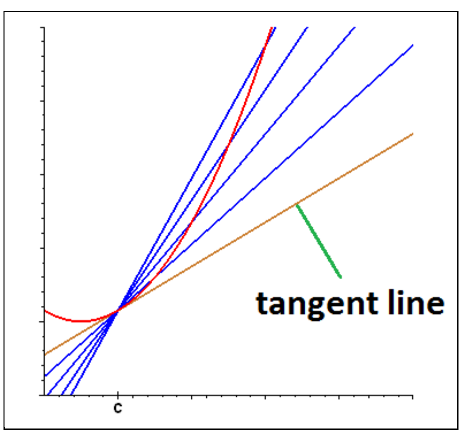
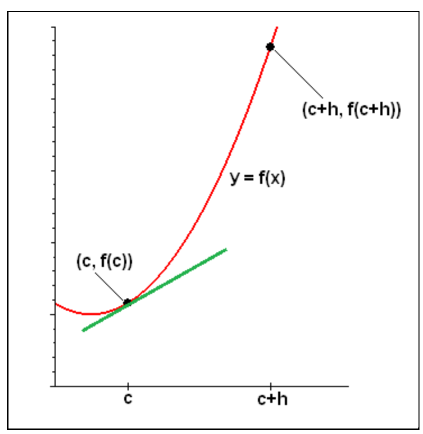


Figure 2.

and talked about the slope of the secant line $m_{\text{sec}} = \dfrac{f(c+h) - f(c)}{h}$. The secant line gives the average rate of change of $f(x)$ from $x = c$ to $x = c + h$. Depending on context, this might be an **average speed over a long trip, an average cost, a country's average debt,** etc. The point here is that <u>two points</u> on the graph are used and we assume a linear (constant) change even if the true graph is not linear. This, in effect, means we are *approximating* the change in the graph and not calculating it exactly—hence the average. Now what happens if we let the two points become one by letting $h \to 0$? Graphically, we get the following image





and algebraically, we get the expression $m_{\tan} = \lim_{h \to 0} m_{\sec} = \lim_{h \to 0} \frac{f(c+h)-f(c)}{h}$. This expression also gives a "slope" but it is the slope of the tangent line to the graph of $f(x)$ at the precise point $(c, f(c))$. This slope is not an average per se because it measures the slope of the curve at <u>one precise location</u>; this is an instantaneous rate of change. This rate of change at $x = c$ is denoted by $f'(c)$ and it is read "f prime of c." It is known as the derivative of the function f. Here is the formal definition.

<u>Definition</u>: The derivative of the function $y = f(x)$ at $x = c$ is given by $f'(c) = \lim_{h \to 0} \frac{f(c+h)-f(c)}{h}$, provided the limit exists. Graphically, the derivative gives the **slope of the tangent line** to the graph of $y = f(x)$ at $(c, f(c))$. Physically, it gives the **instantaneous rate of change** at $x = c$.

Finding $f'(c)$ in different contexts could mean an **instantaneous velocity, a marginal cost,** or **the rate of increase of a country's debt.** Compare this with the earlier phrases of an **average speed over a long trip, an average cost,** or **a country's average debt.**

To sum up, one problem is more **<u>global</u>** in what it provides

$\frac{f(c+h)-f(c)}{h}$
Slope of the secant line
Average rate of change (AROC)
Doesn't incorporate the limit idea

while the other is more **<u>local (specific)</u>** in what it provides

$\lim_{h \to 0} \frac{f(c+h)-f(c)}{h}$ (the derivative)
Slope of the tangent line
Instantaneous rate of change (IROC)
Uses the limit idea