

Finding the equation of a tangent line at a point on a graph... Important takeaway: **The derivative here is a function in its own right** (dependent on  $x$ ). Translation: the slope changes along with the  $x$  value selected (this would be the case for most interesting functions).

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \rightarrow 0} 2x + h$$

$$= 2x$$

$2(-1) = -2 = m$   
 $1 = -2(-1) + b$   
 $1 = 2 + b$   
 $-1 = b$

$y = -2x - 1$

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - (x^2)}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^2 + h^2 + 2xh - x^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{h^2 + 2xh}{h}$$

$$\lim_{h \rightarrow 0} h + 2x \text{ plug in } h=0$$

$f'(x) = 2x$  plug in  $(-1, 1)$   
 $f'(x) = -2 \rightarrow m = -2$   
 $y = -2(x+1) + 1$   
 (Point-slope form)

Picture at above right: Yes,  $f'(x) = 2x$  and then when we choose  $x = -1$ , we can write  $f'(-1) = 2 \cdot (-1) = -2$ . This, again, emphasizes derivative as function.

Examining  $f(x) = \sqrt{x}$ . Similar to the above example, the derivative is a new function,  $f'(x) = \frac{1}{2\sqrt{x}}$ . Note the interpretation of  $f'(0)$  (a slope that can't be calculated since the tangent line is infinitely steep).

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot (\sqrt{x+h} + \sqrt{x})$$

$$\lim_{h \rightarrow 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \rightarrow 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})}$$

$f'(4) = \frac{1}{4}$   
 $f'(0) = \text{undefined}$

$f'(x) = \frac{1}{2\sqrt{x}}$

slope of TL is vertical