MATH 166
Lesson 2.3a
Differentiation Rules

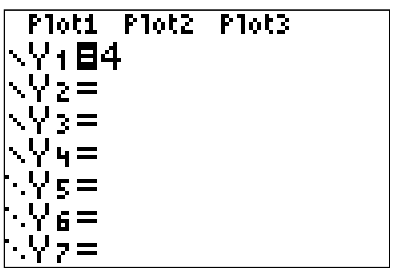
This is a great section! In it, there is one underlying goal: to develop quicker, more efficient ways to find derivatives. In other words, this section is all about *shortcuts*. Throughout this section, we will use the symbol $\frac{d}{dx}$ to indicate the operation of differentiation. We are no stranger to this notation since we may write $\frac{d}{dx}\left[f(x)\right] = f'(x)$.

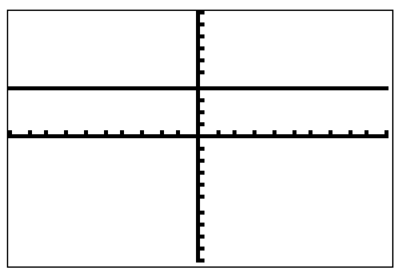
Theorem (Constant Rule): If $f(x) = k$ (where *k* is a constant), then $f'(x) = 0$.

In words, this tells us that the "slope" of any horizontal line is zero. No problem! We already know this.

Example: Find $f'(x)$ given $f(x) = 4$.

Solution: Since $f(x) = 4$, we can conclude that $f'(x) = 0$. See the picture.





This line clearly has a slope of zero.

Theorem (Power Rule): If $f(x) = x^n$ (where *n* is a constant), then $f'(x) = nx^{n-1}$.

In words, this gives us a way to differentiate power functions by placing the power in front of *x* and then dropping the power by one. For example, if $g(x) = x^3$, the above theorem tells us that $g'(x) = 3x^{3-1}$ or $g'(x) = 3x^2$. Let's see if this really checks out.

Example: Use the definition from the previous section to find $g'(x)$ for $g(x) = x^3$.

Solution: Here we go again:

$$\begin{aligned}
g'(x) &= \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\\
&= \lim_{h\to 0}\frac{(x+h)^3-x^3}{h}\\
&= \lim_{h\to 0}\frac{\left(x^2+2hx+h^2\right)(x+h)-x^3}{h}\\
&= \lim_{h\to 0}\frac{x^3+3hx^2+3h^2x+h^3-x^3}{h}\\
&= \lim_{h\to 0}\frac{h\left(3x^2+3hx+h^2\right)}{h}\\
&= \lim_{h\to 0}\left(3x^2+3hx+h^2\right)\\
&= 3x^2.
\end{aligned}$$

Hooray! Indeed, $g'(x)=3x^2$. Notice **how much faster** we can get the answer by using the Power Rule! The time-saving qualities of this theorem cannot be overemphasized.

Theorem (Constant Multiple Rule): If $f(x)=k\cdot g(x)$ (where k is a constant), then $f'(x)=k\cdot g'(x)$.

In words, the constant needs to be "carried along" in the differentiation process.

Theorem (Sum/Difference Rule): Assuming $f(x)$ and $g(x)$ are differentiable functions,

(a) $\frac{d}{dx}\left[f(x)+g(x)\right]=f'(x)+g'(x)$

(b) $\frac{d}{dx}\left[f(x)-g(x)\right]=f'(x)-g'(x)$

In words, the derivative of a sum is the sum of the derivatives; the derivative of a difference is the difference in the derivatives. This theorem extends to as many functions as you wish to consider. With this theorem, we can find derivatives of polynomials as we have come to know them.

We will work out many examples in class. ☺