

Chain Rule (Day 2)

Problem 1a:

Longer way (identifying u explicitly):

Handwritten work for the longer way of Problem 1a. The equation $y = \sin(x^4)$ is circled. Below it, $u = x^4$ is written, with $\frac{du}{dx} = 4x^3$ below that. To the right, $y = \sin(u)$ is written, followed by $\frac{dy}{du} = \cos(u)$. Then, $\frac{dy}{dx} = \cos(u) \cdot 4x^3$ is written, and finally $\frac{dy}{dx} = 4x^3 \cos(x^4)$ is written with an arrow pointing to the final result.

$$y = \sin(x^4)$$

inside $u = x^4$

$$\frac{du}{dx} = 4x^3$$
$$y = \sin(u)$$
$$\frac{dy}{du} = \cos(u)$$
$$\frac{dy}{dx} = \cos(u) \cdot 4x^3$$
$$\frac{dy}{dx} = 4x^3 \cos(x^4)$$

Quicker way:

Handwritten work for the quicker way of Problem 1a. The derivative $y' = \cos(x^4) \cdot 4x^3$ is written in red.

$$y' = \cos(x^4) \cdot 4x^3$$

Problem 1b:

Longer way (identifying u explicitly):

Handwritten work for the longer way of Problem 1b. The equation $y = \sin^4 x = (\sin x)^4$ is circled. Below it, $u = \sin x$ is written, followed by $y = u^4$. Then, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ is written, followed by $= 4u^3 \cdot \cos x$ and finally $= 4\sin^3 x \cos x$ with an arrow pointing to the final result.

$$y = \sin^4 x = (\sin x)^4$$
$$u = \sin x$$
$$y = u^4$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= 4u^3 \cdot \cos x$$
$$= 4\sin^3 x \cos x$$

Quicker way:

Handwritten work for the quicker way of Problem 1b. The derivative $y' = 4(\sin x)^3 \cdot \cos x$ is written in red.

$$y' = 4(\sin x)^3 \cdot \cos x$$

(over)

Big Problem (Level II Chain Rule). Note: Problem (c) is worked out in the pre-class reading.

$$\begin{aligned} \text{a) } y &= \cos^3(2x) \\ y' &= \underline{3(\cos(2x))^2} \cdot \underline{-\sin(2x) \cdot 2} \\ y' &= -6(\cos(2x))^2 \cdot \sin(2x) \end{aligned}$$

$$\begin{aligned} \text{b. } y &= x \tan(7x) \\ y' &= x \cdot \sec^2(7x) \cdot 7 + \tan(7x) \cdot 1 \leftarrow \\ &= 7x \sec^2(7x) + \tan(7x) \end{aligned}$$

$$\begin{aligned} \text{D) } f(x) &= \sin(x) & g(x) &= x+2 \\ f'(x) &= \cos(x) & g'(x) &= 1 \\ h'(x) &= \frac{(x+2) \cdot \cos(x) - \sin(x) \cdot 1}{(x+2)^2} & I(x) &= x^2 \\ & & I'(x) &= 2x \\ h(x) &= \frac{\sin(x)}{x+2} \\ &= \underbrace{2 \left(\frac{(x+2)\cos(x) - \sin(x)}{(x+2)^2} \right)}_{\text{Chain Rule}} \cdot \underbrace{\left(\frac{\sin(x)}{x+2} \right)}_{\text{Product Rule}} \end{aligned}$$