

Last Chain Rule Problem:

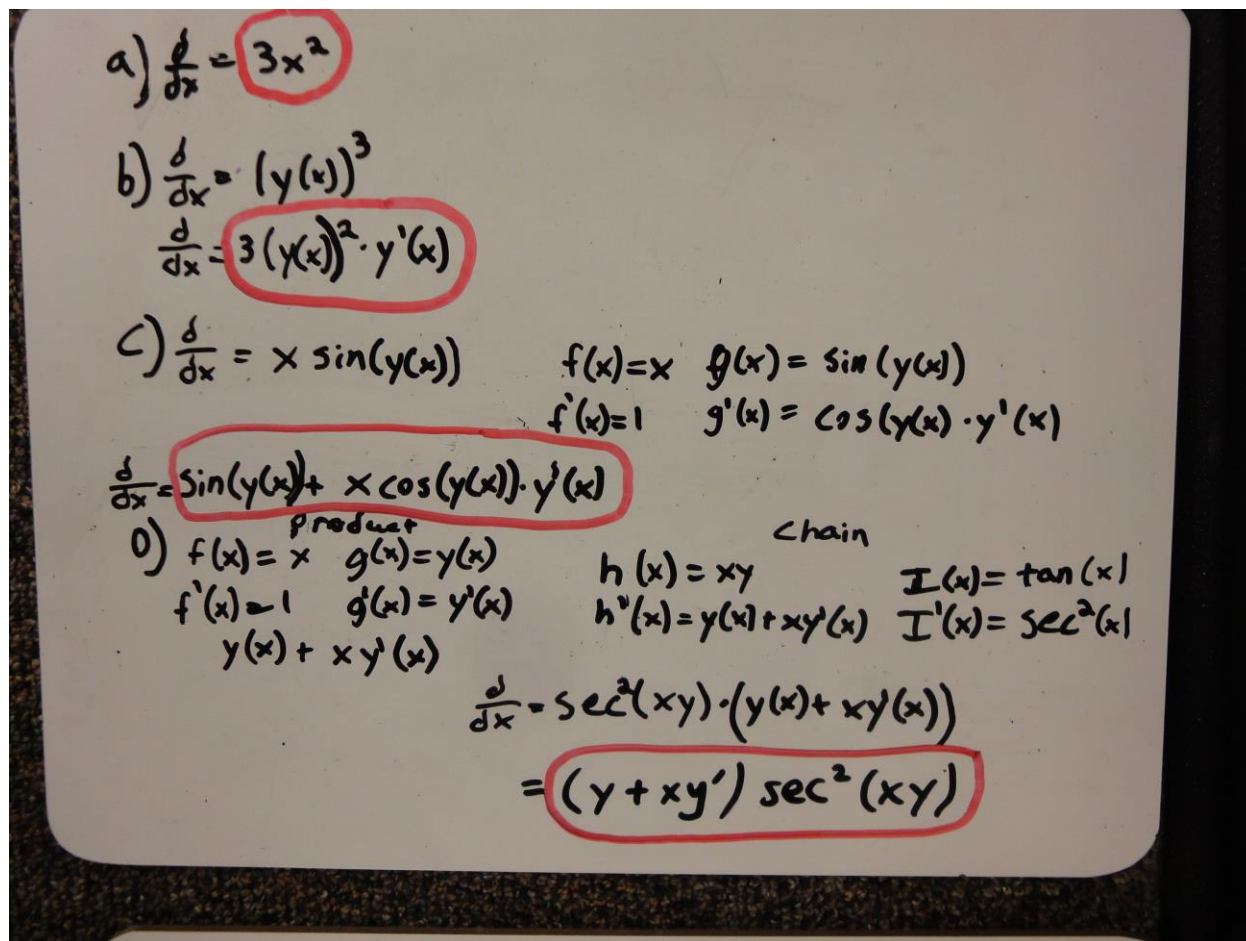
$$\begin{aligned}\frac{f(x)}{g(x)} &= f(x) \cdot [g(x)]^{-1} \\ \frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] &= f(x) \cdot -[g(x)]^{-2} + [g(x)]^{-1} \cdot f'(x) \\ &= f(x) \cdot \frac{-1}{g(x)^2} + \frac{f'(x)}{g(x)} \\ &= \frac{-f(x)}{[g(x)]^2} + \frac{f'(x)}{g(x)} \cdot \frac{g(x)}{g(x)} \\ &= \frac{-f(x) + f'(x)g(x)}{[g(x)]^2} = \frac{f'(x)g(x) - f(x)}{[g(x)]^2}\end{aligned}$$

This one is sooooo close—just missed the Chain Rule.

$$\begin{aligned}\frac{d}{dx} (f(x))(g(x))^{-1} &= \\ (f(x))(-1g(x))^{-2}(g'(x)) + (g(x))^{-1}(f'(x)) &= \\ \frac{-f(x)g'(x)}{(g(x))^2} + \frac{f'(x) \cdot g(x)}{g(x) \cdot g(x)} & \text{Common denom.} \\ \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} &= \end{aligned}$$

This one is spot on. This problem illustrates the validity of the quotient rule by way of using the product (and most important—chain) rules.

Warm Up for implicit differentiation:



A brief note on notation: It is mathematically correct to write  $\frac{d}{dx}(x^3) = 3x^2$  or if  $y = x^3$ , we can say  $\frac{dy}{dx} = 3x^2$ . However, it's not correct to say something like  $\frac{d}{dx} = 3x^2$ . This is because  $\frac{d}{dx}$  is an **operator** so it must operate on *something* (usually an expression of some sort). Correct answers are circled above.