

Problem 1 (done in class). What is neat about this one is you can conclude that the radial line and the tangent line are perpendicular (they meet at a ninety-degree angle). That is,

$$m_{\text{radial line}} \cdot m_{\text{TL}} = \frac{y}{x} \cdot \left(-\frac{x}{y}\right) = -1. \text{ This is the geometric interpretation.}$$

Problem 2. See below.

The image shows handwritten work on a piece of paper. The steps are as follows:

$$x^2y + xy^2 = 1$$

$$\frac{d}{dx}(x^2y + xy^2) = \frac{d}{dx}(1)$$

$$y \cdot 2x + x^2 \frac{dy}{dx} + y^2 + (x \cdot 2y \frac{dy}{dx}) = 0$$

$-y(2x)$ $-y^2$

$$x^2 \frac{dy}{dx} + x \cdot 2y \frac{dy}{dx} = -y^2 - 2xy$$

$$\frac{dy}{dx} (x^2 + 2xy) = \frac{-y^2 - 2xy}{x^2 + 2xy}$$

$$\frac{dy}{dx} = \frac{-y^2 - 2xy}{x^2 + 2xy}$$

Problem 3. (Also done in class). What is neat about this one is we obtain a derivative for a function we have previously studied in trigonometry. $\sin y = x$ translates to

$$y = \sin^{-1} x = \arcsin x \text{ and we found its derivative to be } \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}.$$