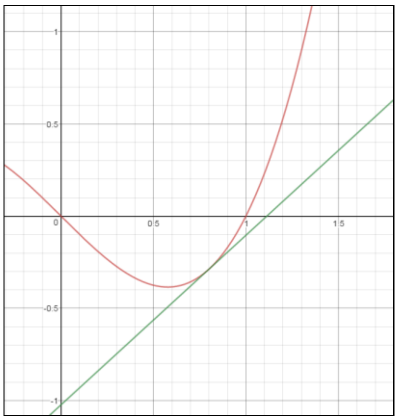
MATH 166
Lesson 2.7
Applications

The idea of derivative, despite being a single object $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, has many interpretations depending on the context in which it is used. **Geometrically**, we have seen the derivative as the slope of the tangent line to the graph of $y = f(x)$.



Physically, if $s(t)$ describes the position of a particle moving along a straight line, we have interpreted the derivative $s'(t)$ as the (instantaneous) velocity of the particle. We often write $v(t) = s'(t)$. There is also a physical interpretation for the second derivative $s''(t)$. This gives the rate of change of the velocity and is well-known as acceleration. We'll use the notation $a(t)$. To summarize,

- $s(t)$ indicates the position of the object.
- $v(t)$ represents the velocity and can be determined via $v(t) = s'(t)$.
- $a(t)$ gives the acceleration of the object and can be found by $a(t) = s''(t)$ or, equivalently, $a(t) = v'(t)$.

Two more interpretations of the derivative follow.

Growth Rate

Most of the changes in our world can be described in terms of growth. You can think of things such as prices, population, investments, and the like. Describing growth allows us to make predictions into the future. For example, suppose $p = p(t)$ describes the population of foxes in a preserve (*t* is time in years). If we look at the **average growth**

rate from time $t = a$ to $t = a + h$, we would get $\frac{\Delta p}{\Delta t} = \frac{p(a+h) - p(a)}{h}$. As $h \to 0$, this average growth rate becomes the **instantaneous growth rate** $\frac{dp}{dt} = p'(t)$ (sometimes just called the **growth rate**). If we calculated a quantity such as $p'(2016) \approx 25$, we would interpret this as a growth rate of about 25 foxes/year. This provides useful information to animal welfare organizations who wish to ensure the habitat can sustain a growing population of foxes.

Economics & Business

Cost functions are popular in business and economics as the cost $C = C(x)$ depends on the number of items produced x. It is common for cost functions to take the form $C(x) = 200 + 0.1x$ or something similar. The \$200 represents a fixed setup cost whereas the \$0.10 represents a variable cost per item produced. **Average cost** is defined by $y = \frac{C(x)}{x}$ (this is the cost divided by the number produced) and a reasonable assumption is that the average cost drops as the number of items produced increases. While the average cost looks to the past (items already produced), you might wonder about the cost of producing more items. We know that we can define the derivative $C'(x) = \lim_{h \to 0} \frac{C(x+h) - C(x)}{h}$ but what does it mean here? Although $h \to 0$, we can simply substitute a small number for *h* such as $h = 1$ and we would end up with $C'(x) \approx \frac{C(x+1) - C(x)}{1} = C(x+1) - C(x)$. What this statement says is that the derivative as we know it $C'(x)$ gives us a good estimate of the cost incurred by producing an additional item: $C(x+1) - C(x)$. If this cost is one the company cannot absorb, then it would not make sense to continue production beyond *x* items. If, however, this cost is low, this is very useful information to have on hand. $C'(x)$ is widely known in business as the **marginal cost**—the approximate cost to produce an additional item after producing *x* items.

The list goes on and on….

1. Physicists and engineers are interested in the rate of flow of a liquid through a pipe (think of analogies such as water flowing through a conduit, blood flowing through an artery, etc.)
2. Forecasters examine how pressure in the atmosphere changes with altitude.
3. Even in places where you would least expect it: How fast does a rumor spread? How quickly is a popular meme shared? These are all questions about rate of change.