

Multiple Choice Question (you need the Product Rule and the Chain Rule):

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{\pi}{3} \left( r^2 \frac{dh}{dt} + h 2r \frac{dr}{dt} \right)$$

$$\frac{dV}{dt} = \frac{\pi}{3} \left( 2r \frac{dr}{dt} h + r^2 \frac{dh}{dt} \right)$$

A.

$f(x) = r^2$	$g(x) = h$	$f(x) = r$	$g(w) = x^2$
$f'(x) = 2r \frac{dr}{dt}$	$g'(x) = \frac{dh}{dt}$	$f'(x) = \frac{dr}{dt}$	$g'(w) = 2x$
		$2(r) \frac{dr}{dt}$	← Chain

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = \frac{1}{3}\pi \left( 2r \frac{dr}{dt} h + r^2 \frac{dh}{dt} \right)$$

A.

Boat Problem (Part I):

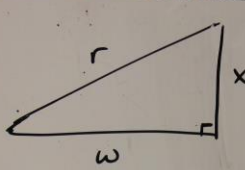
$$(x^2) + w^2 = r^2$$

↑  
constant

$$2w \frac{dw}{dt} = 2r \frac{dr}{dt}$$


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$$\frac{dw}{dt} = \frac{r}{w} \frac{dr}{dt}$$



$$w^2 + \underbrace{x^2}_{\text{constant}} = r^2$$

$$\frac{d}{dt} (w^2 + x^2) = \frac{d}{dt} (r^2)$$

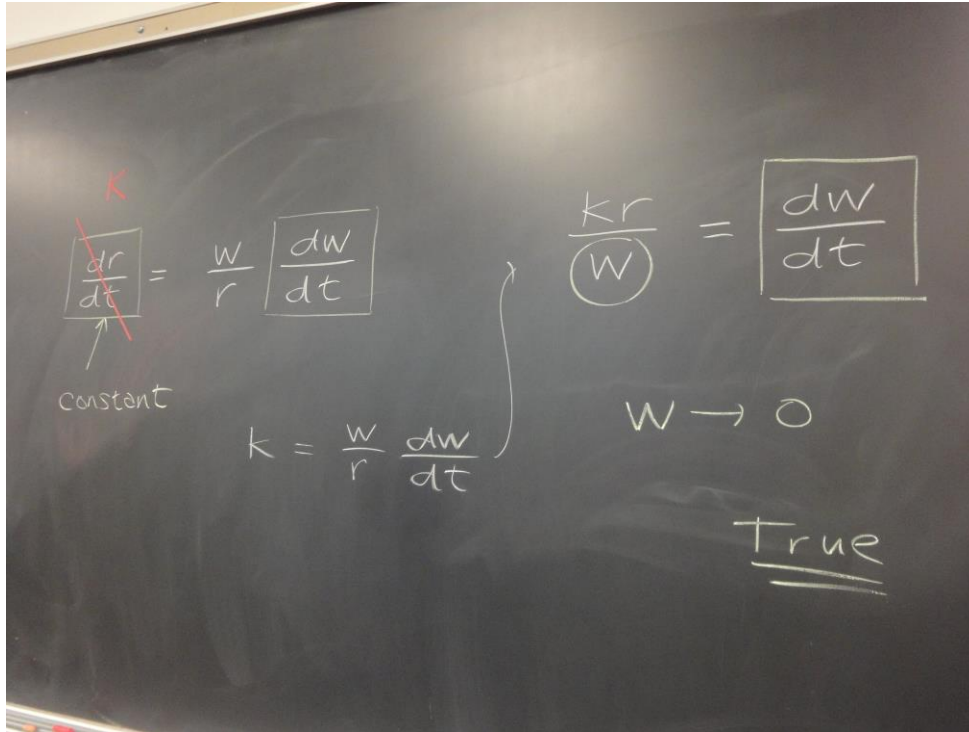
$$\cancel{2}w \frac{dw}{dt} = \cancel{2}r \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{w}{r} \frac{dw}{dt} \quad \text{C}$$

not constant

The above work is equivalent.

Boat Problem (Part II):



Note: As  $w \rightarrow 0$ ,  $\frac{dw}{dt}$  grows in magnitude. In other words, the boat moves faster as it gets closer to the dock.