

Min/Max problems on a closed interval. Here are the solutions. Problem (c) was done for you in class.

(A) $f(x) = 1 + 3x^4 - 4x^3$ on $[-1, 2]$
 $f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

x	f(x)
-1	8
0	1
1	0 min
2	17 max

$0 = 12x^2$
 $x = 0$
 $0 = x - 1$
 $x = 1$

(B) $g(x) = x^{4/5}$ on $[-32, 32]$
 $g'(x) = \frac{4}{5}x^{-1/5} = \frac{4}{5\sqrt[5]{x}} \neq 0$
 $g'(x)$ is undefined at $x = 0$

x	g(x)
32	16 max
0	0 min
-32	16 max

Important information for problem (d)... $x = 0$ appears to be an important number to check in the table (since it is not in the domain of the function). However, $x = 0$ falls *outside* of the window $[1/5, 4]$ so we can omit it from analysis. Similarly, $x = -1$ makes the derivative zero but we exclude this value because it is not in the interval $[1/5, 4]$.

(D) $Z(x) = x + \frac{1}{x}$ on $[\frac{1}{5}, 4]$
 $Z'(x) = 1 - \frac{1}{x^2}$

$0 = 1 - \frac{1}{x^2}$
 $-1 = -\frac{1}{x^2}$
 $x^2 = 1$
 $x = 1$

x	Z(x)
$\frac{1}{5}$	5.2 max
4	4.25
1	2 min
0	UNDEF

$x = \pm 1$ ← $x = -1$ Not in interval

E) $y = \frac{\cos x}{2 + \sin x}$ on $[0, 2\pi]$

$$y' = \frac{(2 + \sin x)(-\cos x) - \cos x(\cos x)}{(2 + \sin x)^2}$$

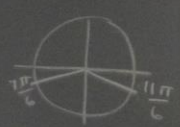
$$= \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2 + \sin x)^2}$$

$$= \frac{-2\sin x - \overset{=1}{\sin^2 x + \cos^2 x}}{(2 + \sin x)^2}$$

$$= \frac{-(2\sin x + 1)}{(2 + \sin x)^2} \rightarrow \text{set } = 0$$

\rightarrow Never undefined

$$-2\sin x - 1 = 0$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = x$$


$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

x	f(x)
$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{3}$ min
$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{3}$ max
0	$\frac{1}{2}$
2π	$\frac{1}{2}$

A graph of the above function confirms that the maximum occurs near the right endpoint of the interval (notice that $\frac{11\pi}{6}$ falls just short of 2π) whereas the minimum occurs almost in the middle of the interval ($\frac{7\pi}{6}$ is just a hair larger than π). See the graph of $y = \frac{\cos x}{2 + \sin x}$ on the interval $[0, 2\pi]$:

```
NORMAL FLOAT AUTO REAL RADIAN MP
WINDOW
Xmin=0
Xmax=6.283185307
Xscl=1
Ymin=-.5773502692
Ymax=√(3)/3
Yscl=1
Xres=1
ΔX=.02379994434469
TraceStep=.04759988868939
```

