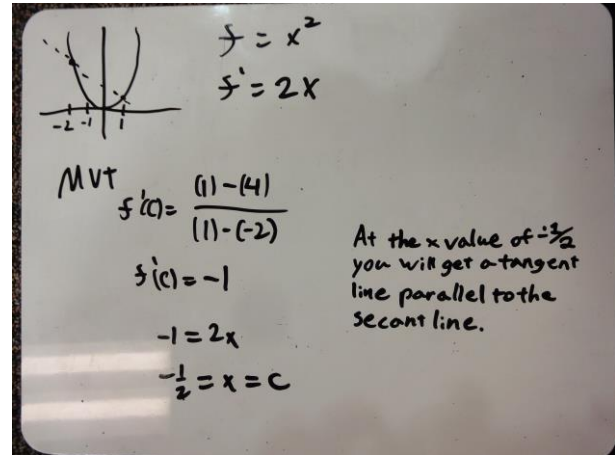
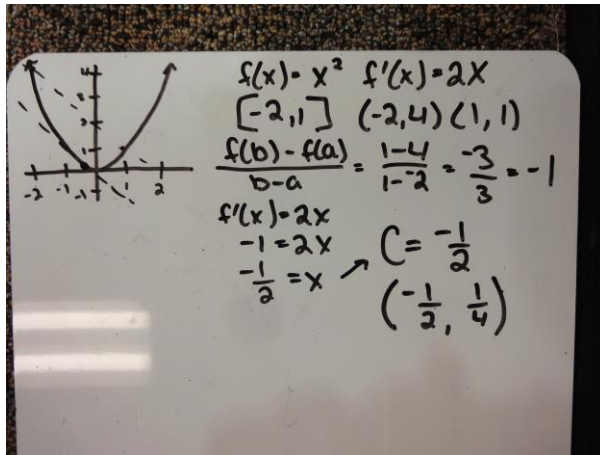


Mean Value Theorem

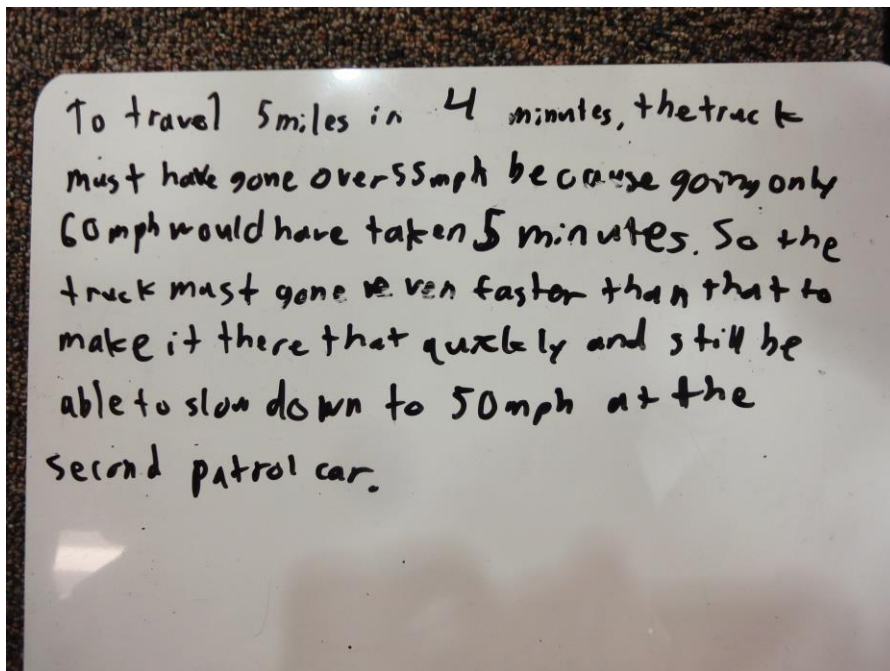
Graphical problem:



At $c = -\frac{1}{2}$, the slope of the tangent line to $f(x) = x^2$ is the same as the slope of the secant line through the endpoints $(-2, 4)$ and $(1, 1)$. See the above whiteboards.

Proving that the truck driver was speeding....

This captures the time factor; the driver was definitely speeding:



But how fast was s/he going?

$$\frac{55 \text{ mph}}{60 \text{ mins}} = .917 \text{ miles per minute}$$
$$.917 \text{ miles per minute} \cdot 4 \text{ minutes}$$
$$= 3.67 \text{ miles}$$

is the farthest that he could have traveled in the 4 minutes by going the speed limit

So...

$$\frac{5 \text{ miles}}{4 \text{ minutes}} = 1.25 \text{ miles per minute}$$
$$1.25 \text{ miles per minute} \cdot 60 \text{ mins}$$
$$= 75 \text{ mph}$$

Average speed for 5 miles in 4 min.

The driver traveled 5 miles in just 4 minutes which amounts to an average speed of 75 miles/hour. This 75 mph corresponds to the slope of the secant line because this is an **average rate of change**. By the MVT, we know there must be a point somewhere between the patrol cars where the truck driver was traveling exactly 75 mph (this corresponds to the tangent line or **instantaneous rate of change**). We do not know *where* it happened but we do know it definitely happened (the value of c is sometimes impossible to locate exactly).