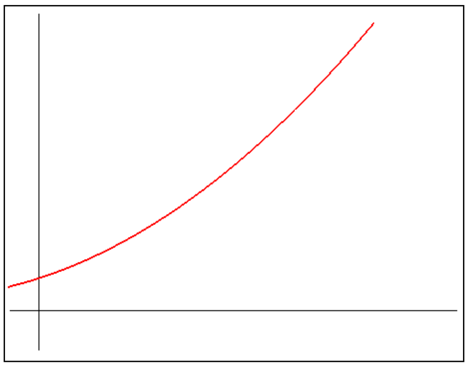
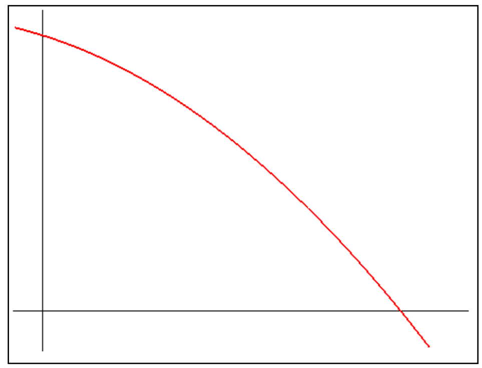
MATH 166
Lesson 3.3a
Increasing and Decreasing Functions; First Derivative Test

Most people are familiar with what is meant by saying "a function is increasing" or "a function is decreasing." A function is increasing if it is *rising* from left to right; a function is decreasing if it is *falling* from left to right. Take a look at the pictures below.



INCREASING



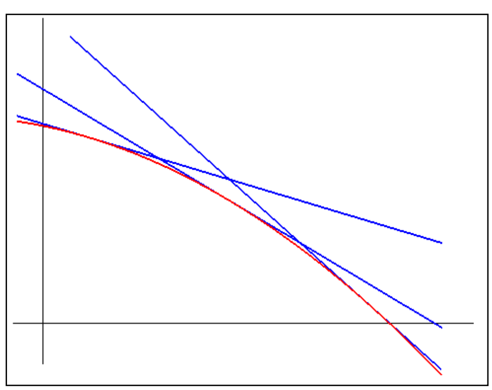
DECREASING

Let's make this more precise; here is the very same definition from a Calculus perspective.

<u>Definition</u>: Let f be continuous on $[a,b]$ and differentiable on (a,b).

1. If $f'(x) > 0$ for all x in (a,b), then f is **increasing** on $[a,b]$.
2. If $f'(x) < 0$ for all x in (a,b), then f is **decreasing** on $[a,b]$.
3. If $f'(x) = 0$ for all x in (a,b), then f is **constant** on $[a,b]$.

Let's talk about one of these statements in detail. For example, the second statement says that *if the slope of the tangent line is negative*, then the function is decreasing there. Take a closer look at this:



If you study this carefully, you can see that negative derivatives (that is, $f'(x)<0$) imply that the function is decreasing. Notice that all of the tangent lines have negative slope—meaning that the function is falling (decreasing) from left to right. You can support the other statements in the definition with similar pictures. From this, we gather that the **sign of the derivative** (positive or negative) gives the information about where the function is increasing or decreasing.

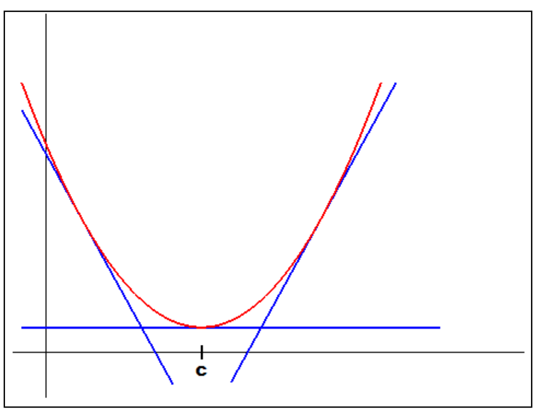
Here is one test that helps us identify the min's and max's of functions. It does this by examining carefully where the function is increasing and decreasing.

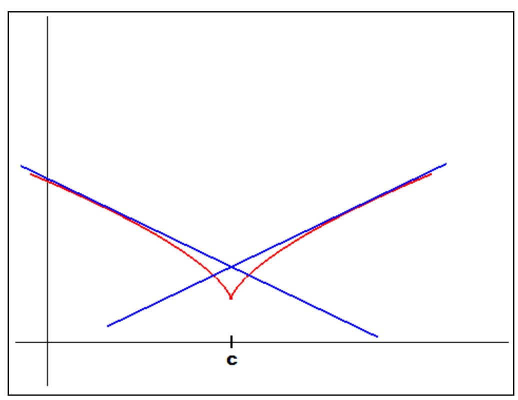
First Derivative Test

Let c be a critical number of f. Also, let f be differentiable on an interval containing c (but f may or may not be differentiable at $x=c$).

1. If f' changes sign from negative to positive at $x=c$, then $f(c)$ is a **relative minimum**.
2. If f' changes sign from positive to negative at $x=c$, then $f(c)$ is a **relative maximum**.

This is all about looking at the sign of $f'(x)$. Let's look at statement 1 in detail. The slope of the tangent line changes from negative to positive and $x=c$ is a critical number. Here are two possible pictures that fit this situation:





In both cases, notice that $(c, f(c))$ corresponds to a relative minimum. In the first picture, $f'(c)=0$; in the second picture, $f'(c)$ fails to exist. This fits the earlier idea of a critical number to a function. You can think about similar pictures for the second part of the First Derivative Test.

We will look at an example in class!