

## Increasing/Decreasing + A little bit of Concavity

Three Problems on intervals of increase/decrease and extreme values:

(A)  $f(x) = x^2 - 4x$  ←  
 $f'(x) = 2x - 4 = 0$   
 $x = 2$   
 $* f(2) = -4$   
 $(2, -4)$  is a Min  
 decreasing on  $(-\infty, 2)$   
 increasing on  $(2, \infty)$

We recognize this as a parabola.

(C)  $y = x\sqrt{6-x} = x(6-x)^{1/2}$   
 $\rightarrow y' = (6-x)^{-1/2} + (6-x)^{1/2}(-1)$   
 $* = \frac{-x}{2(6-x)^{3/2}} + (6-x)^{1/2}$   
 $= \frac{2(6-x) - x}{2(6-x)^{3/2}} = \frac{12-3x}{2(6-x)^{3/2}} \leftarrow x-4$   
 $\leftarrow \text{undef. @ } x=6$   
 inc.  $(-\infty, 4)$   
 dec.  $(4, 6)$   

x	y
0	0
4	4√2 Max
6	0 Min

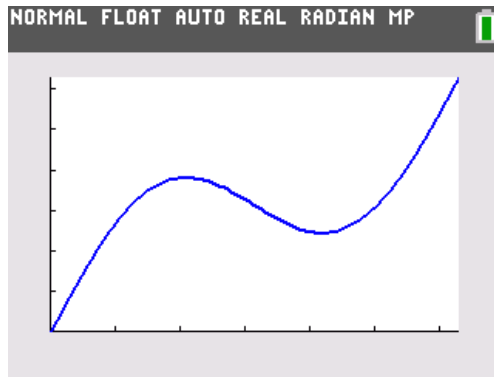
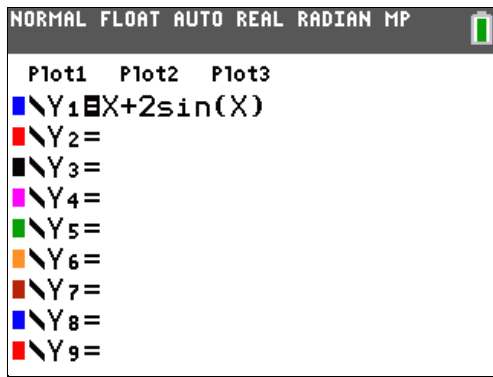
The analysis above is a bit more challenging.

This one involves a good dose of trigonometry:

(D)  $h(x) = x + 2\sin x$ ,  $[0, 2\pi]$   
 $h'(x) = 1 + 2\cos x$   
 $0 = 1 + 2\cos x$   
 $\frac{-1}{2} = \cos x$   
 $\cos^{-1} \frac{-1}{2} = \frac{2\pi}{3}$  or  $\frac{4\pi}{3}$

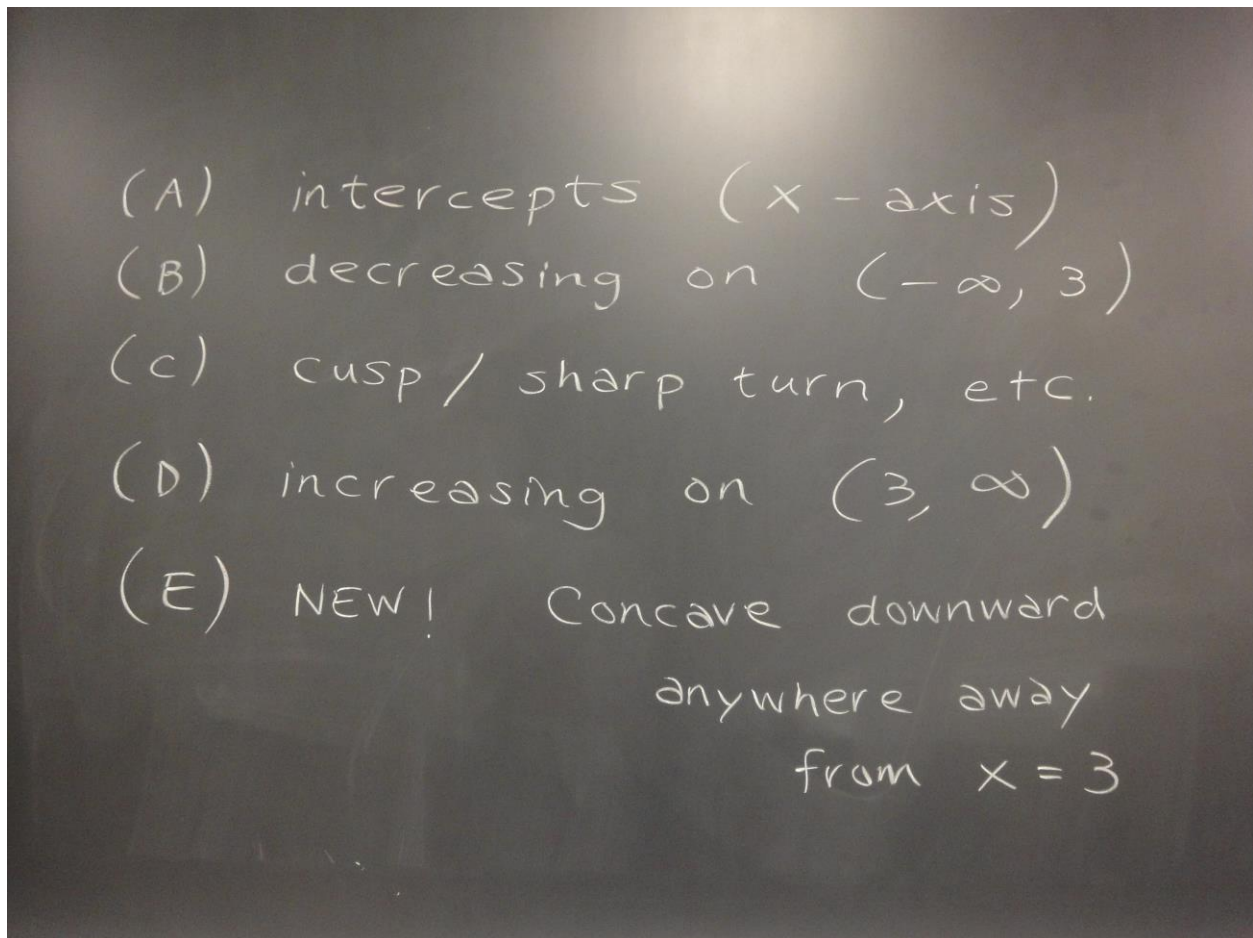
x	y	Notes
0	0	Abs Min
$\frac{2\pi}{3}$	3.826	rel. max
$\frac{4\pi}{3}$	2.457	rel. min
$2\pi$	6.283	Abs Max

Looking at the graph also helps confirm the relative min and max (direct from the critical numbers) and the absolute min and max (from the endpoints). See the top of the next page.

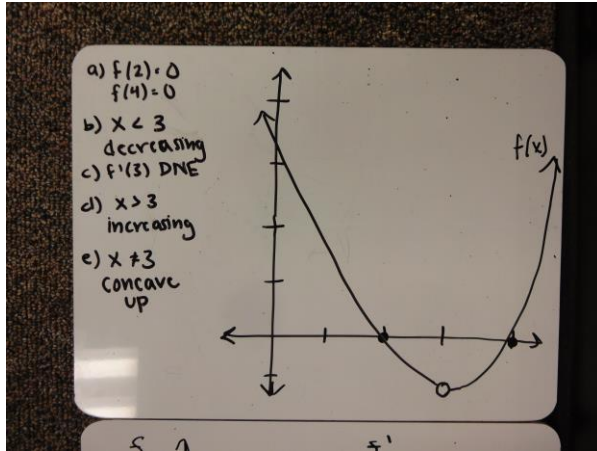


### Constructing a graph from given information.

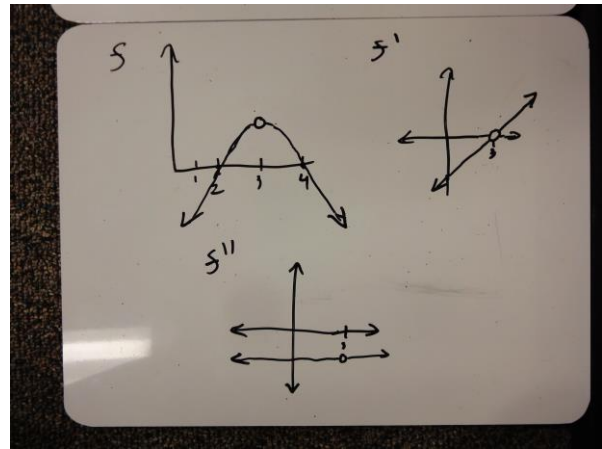
First, it is a good idea to translate each of the statements. That is, what does each statement tell you about the graph?



Here are some examples that are partially correct:

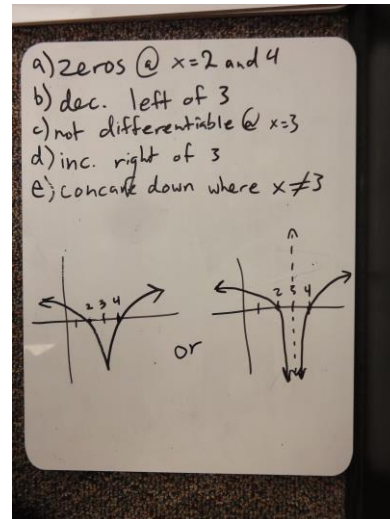
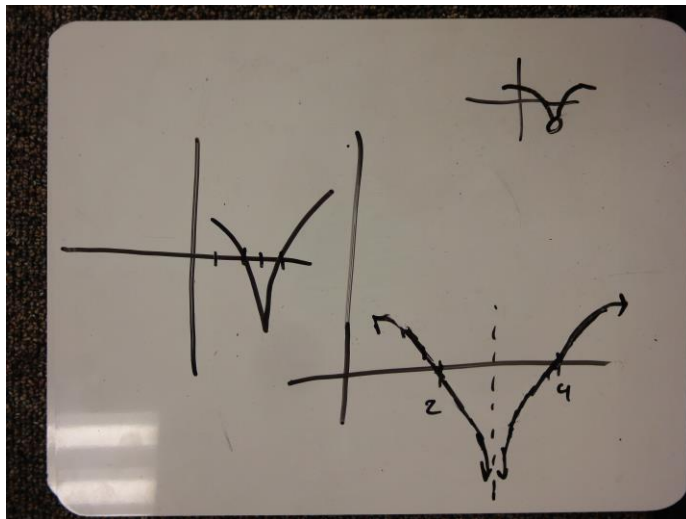


Only Condition E is not satisfied above.



Condition B and D are not satisfied.

These examples satisfy all five (A-E) of the properties:



And this one (see the next page) shows the progression of trying to incorporate each additional property while making sure we don't lose a property that was previously satisfied. Great thinking!!

