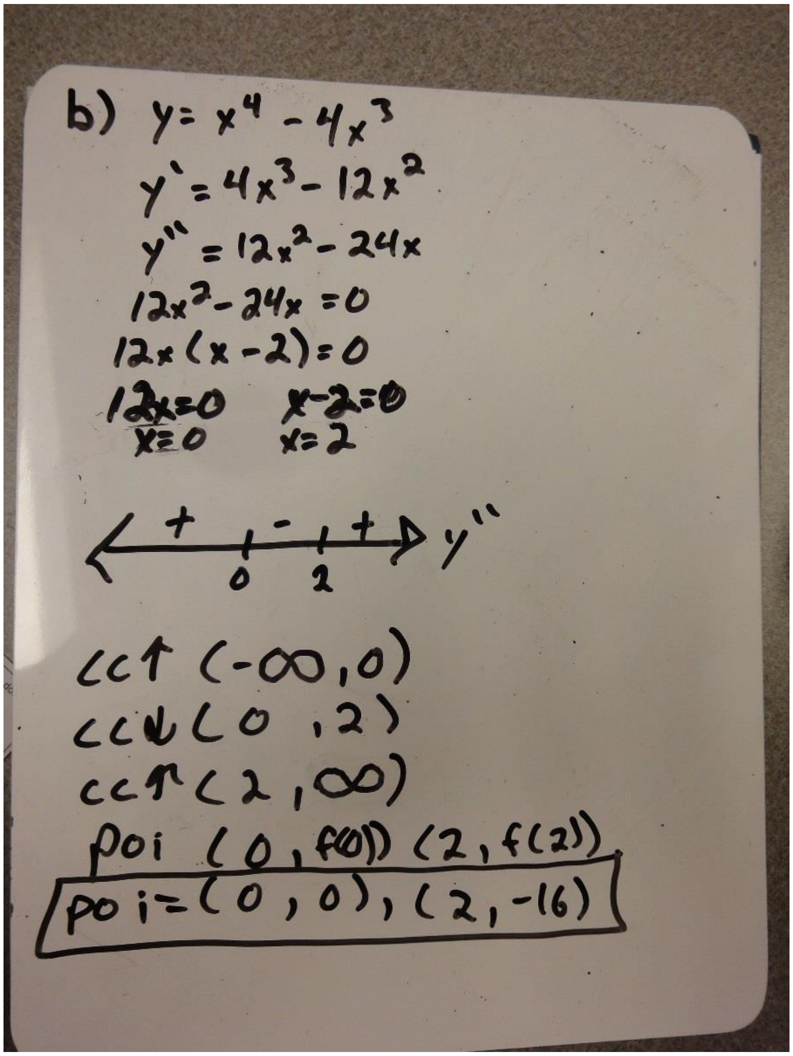
Concavity Problem:



The graphing calculator should support all of this (the graph is concave upward followed by a small section of concave downward followed still by a return to concave upward).

Second Derivative Test (see next page):

Analysis of $y = x + \frac{1}{x}$ w/ SDT

Note: $x = 0$ (VA) Also, as $x \to \infty$

$y = x + \frac{1}{x}$ $\left(\frac{1}{x} \to 0\right)$ so

$y = x + \frac{1}{x} \approx x$ (behaves like $y = x$)

$y' = 1 - \frac{1}{x^2}$

$= \frac{x^2 - 1}{x^2}$

$= 0$

↑ set

$x = \pm 1$ critical #s

$x = 0$ (not in domain)

Now test critical #s into $y'' = \frac{2}{x^3}$

$y''(1) = \frac{2}{1^3} = 2 > 0$

$y''(-1) = \frac{2}{(-1)^3} = -2 < 0$

$y''(1) > 0 \Rightarrow$ CC ↑

↑ $x = 1$ corresponds to a min

$y''(-1) < 0 \Rightarrow$ CC ↓

↑ $x = -1$ corresponds to a max

$(1, y(1)) = (1, 2) \leftarrow$ MIN

$(-1, y(-1)) = (-1, -2) \leftarrow$ MAX

$y' = 1 - x^{-2}$

$y'' = 2x^{-3}$

$= \frac{2}{x^3}$

$x = 0$ (again, not in domain)

$\overset{-}{\longleftarrow} \underset{0}{+} \overset{+}{\longrightarrow}$ sign of y''

CC↓ CC↑

concave ↓ : $(-\infty, 0)$

concave ↑ : $(0, \infty)$

No inflection point at $x = 0$!!

(vertical asymptote)