

Determining $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x})$ analytically:

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x}) \cdot \frac{x + \sqrt{x^2 + x}}{x + \sqrt{x^2 + x}} = 1$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 - (x^2 + x)}{x + \sqrt{x^2 + x}} = 1$$

$$= \lim_{x \rightarrow \infty} \frac{-x}{x + \sqrt{x^2 + x}} = \frac{-\infty}{\infty}$$

$$\begin{aligned} \sqrt{x^2 + x} &= \sqrt{x^2 \left(1 + \frac{1}{x}\right)} \\ &= \sqrt{x^2} \sqrt{1 + \frac{1}{x}} \\ &= x \sqrt{1 + \frac{1}{x}} \end{aligned}$$

And the strong finish:

$$\lim_{x \rightarrow \infty} \frac{-x}{x + x \sqrt{1 + \frac{1}{x}}} \quad \begin{array}{l} \div x \\ \div x \end{array}$$

$$= \lim_{x \rightarrow \infty} \frac{-1}{1 + \sqrt{1 + \frac{1}{x}}}$$

$$= \frac{-1}{1 + \sqrt{1}} = \boxed{-\frac{1}{2}}$$