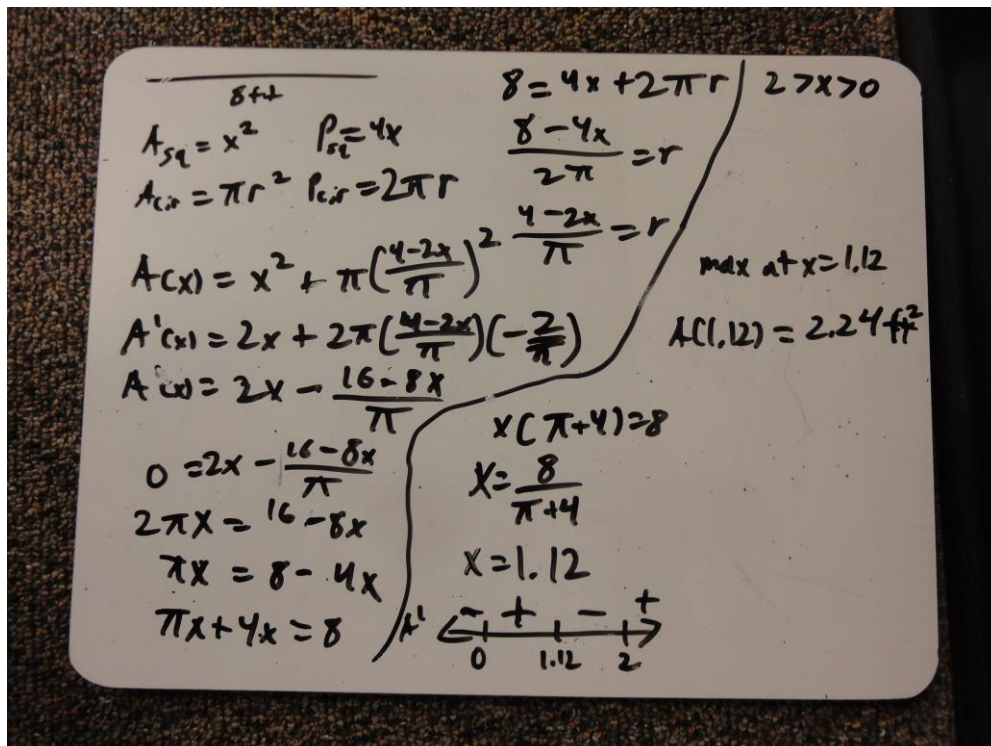


More on optimization (Level II Problem):

Eight feet of wire is to be used to form a square and a circle. How much of the wire should be used for each figure to enclose the maximum area?

For discussion purposes, use x to denote the side of the square and r for the radius of the circle. Show that the area, as a function of x , is given by $A(x) = x^2 + \pi \left(\frac{4-2x}{\pi} \right)^2$.

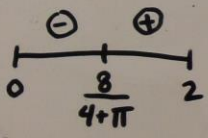
The mathematics on this board (below) is valid but the conclusion is not. It turns out that $x \approx 1.12$ corresponds to a minimum, not a maximum (but the critical number identified is correct).



Here is a great solution:

$P = 8$
 $P = 4x + 2\pi R$
 $8 = \frac{2}{4}x + 2\pi R$
 $4 - 2x = \pi R$
 $R = \frac{4 - 2x}{\pi}$

$A = x^2 + \pi R^2$ (maximize)
 $= x^2 + \pi \left(\frac{4 - 2x}{\pi} \right)^2$
 $= x^2 + \frac{\pi(4 - 2x)^2}{\pi^2}$
 $= x^2 + \frac{4x^2 - 16x + 16}{\pi}$
 $= x^2 + \frac{4}{\pi}x^2 - \frac{16}{\pi}x + \frac{16}{\pi}$
 $= \frac{4 + \pi}{\pi}x^2 - \frac{16}{\pi}x + \frac{16}{\pi}$
 $A' = \frac{2(4 + \pi)}{\pi}x - \frac{16}{\pi} = 0$
 $\frac{2(4 + \pi)}{\pi}x = \frac{16}{\pi}$
 $(4 + \pi)x = 8$
 $x = \frac{8}{4 + \pi} = 1.12$

$0 < x < 2$

 $\frac{8}{4 + \pi} \neq \text{max.}$

x	A(x)
0	5.09
$\frac{8}{4 + \pi}$	2.24
2	4

$R = \frac{4}{\pi}$ (only make circle)

The chart in the lower left of the board shows that the maximum area occurs (5.09 ft²) when $x = 0$. In other words, **don't use any wire to make the square**; just make the circle. This makes sense if you think about it...circles enclose large areas without using a lot of material to enclose the circle (i.e., circles have no corners).