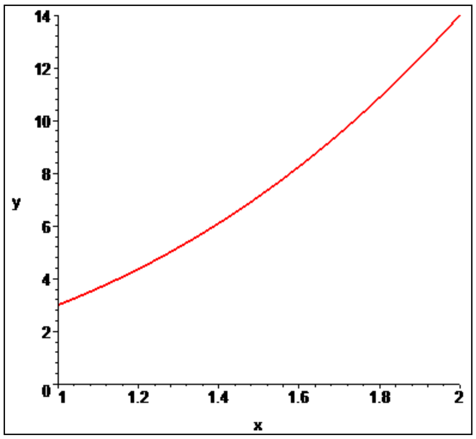
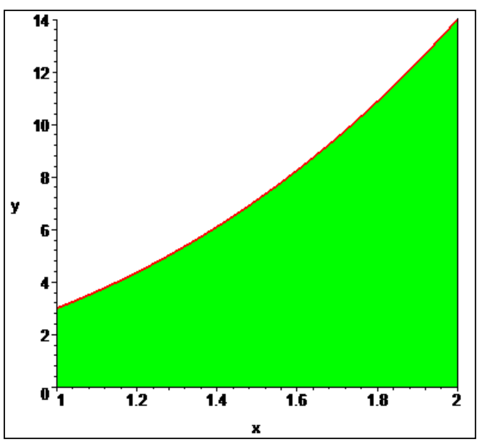
MATH 166
Lesson 4.1
Areas & Distances

In this unit, we will study a problem of significant importance in the development of Calculus—that of finding the area under a curve. For example, take a look at the function below on the window $[1, 2, 0, 14]$:

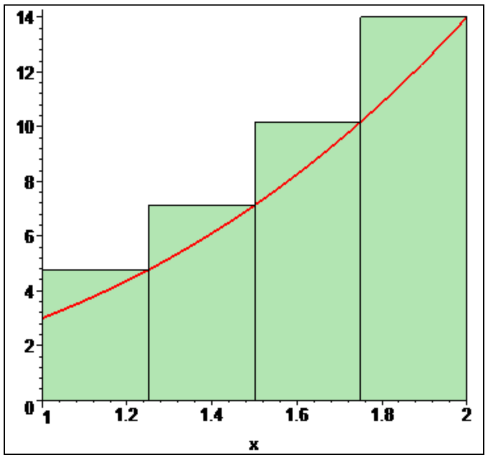


Suppose you wished to calculate the area under the curve between the two vertical lines $x = 1$ and $x = 2$ and above the *x*-axis. In other words, suppose you were trying to calculate the shaded area below:

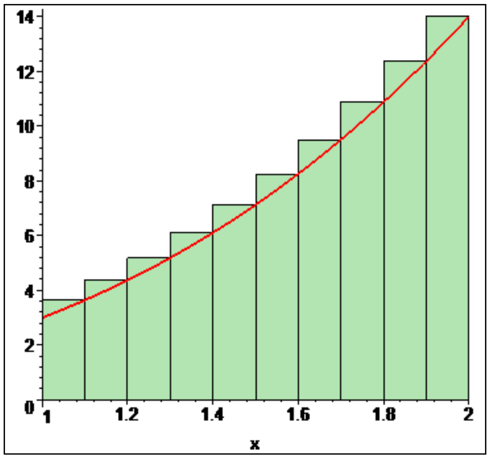


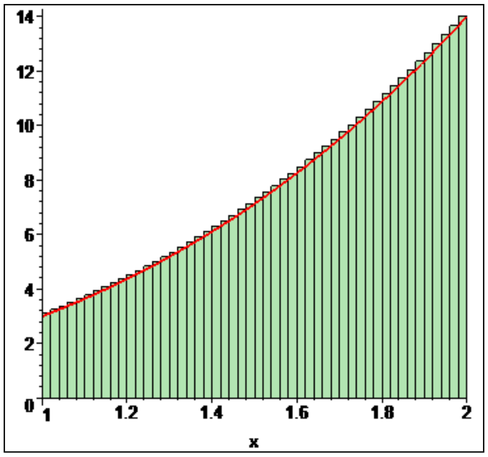
If you think this is a challenging problem . . . you are absolutely right! Areas of certain shapes are easy to compute (including squares, rectangles, circles, trapezoids, parallelograms, regular polygons, etc.). This is because Geometry has developed nice convenient formulas that compute these areas efficiently. However, the area shaded above is an *irregular* shape; it cannot be classified as any "familiar" object. If you still want its exact area, we have to use a different method. The method discussed in this section will approximate such areas with higher and higher degrees of accuracy. Eventually, we'll be able to find the exact area. This is the *marvel* that is Calculus. We will see the connection to **limits** almost right away. As we progress deeper into this unit, it will then be revealed how the area problem is related to **differentiation**, thus tying together most of what we have learned.

Here is the basic idea that will be used in this unit. Using a fixed number of rectangles (say, $n = 4$), we can *estimate* the area under the curve by using the following picture:



If we compute the areas of the four rectangles, this will give us an approximation for the area under the curve but two notes are in order here. First, finding the area of a rectangle is easy—just length times width. Once we find the four areas as seen above, just add them up and there's your approximation. The other note is that this approximation isn't very accurate. Adding the areas of the four rectangles overestimates the true area under the curve. Therefore, there is a clear motivation to try a larger *n* value (say, $n = 10$, see graph at the lower left) or something even larger (say, $n = 50$, see graph at the lower right).





At this point, you may be able to predict how limits are connected to the area problem. If we let *n* grow, our approximations get better and better. We know that for $n \to \infty$ our approximation will become an *exact* value—precisely what we're after in this section.