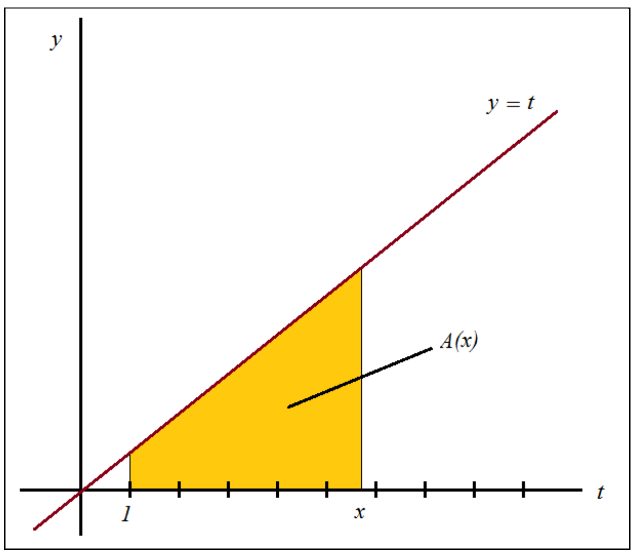
MATH 166
Lesson 4.3a
Fundamental Theorem of Calculus, Part I

So begins the first half of the most significant content we cover in this course. The first thing to do is to introduce the idea of an accumulation function, $A(x)$. An *accumulation function* keeps a running tally of the area under a curve starting at some fixed point a and terminating at some point x. For example, consider this accumulation function:

$$A(x)=\int_1^x t\,dt.$$

This function tracks the area under the curve $f(t)=t$ from $t=1$ to $t=x$. See the diagram:

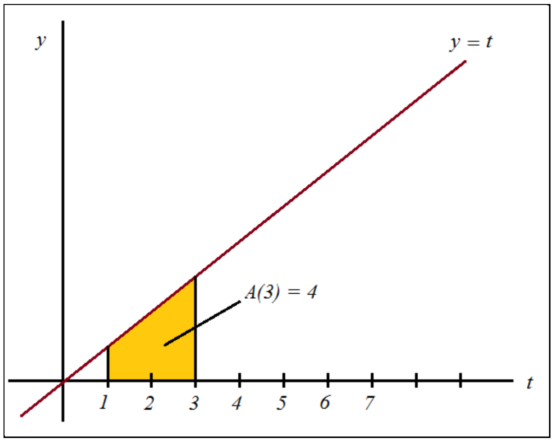


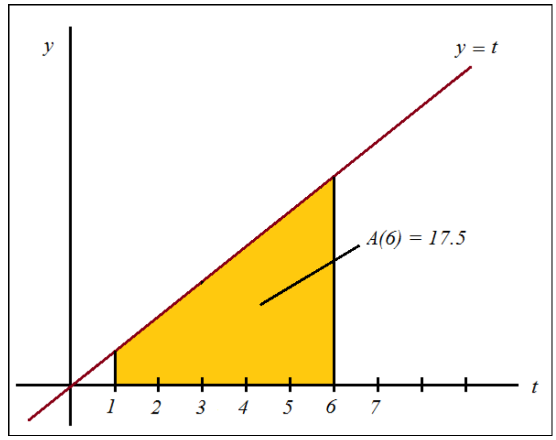
Note: It is commonplace to use *t* as the "*x*-axis" since *x* has a concrete meaning here—*x* is the upper limit of integration in the accumulation integral.

Notice that the accumulation function, like most any function, can take on different values depending on your choice of inputs. For example,

$A(3)=\int_1^3 t\,dt$ $=$ area from $t=1$ to $t=3$ $=4$ square units	and	$A(6)=\int_1^6 t\,dt$ $=$ area from $t=1$ to $t=6$ $=17.5$ square units

Pictures confirm this:





In the more general setting, we have the accumulation function $A(x) = \int_a^x f(t)\,dt$ with a picture like this (see **Figure 1**):

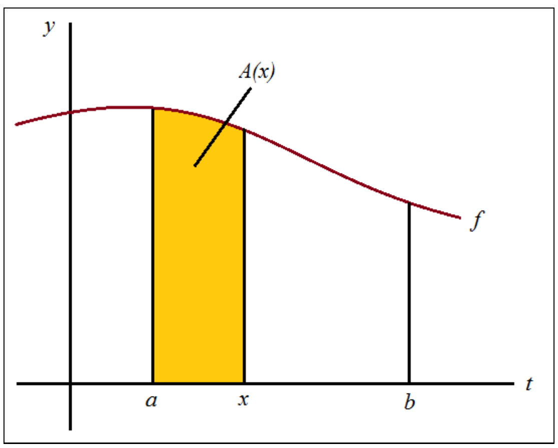


Figure 1.

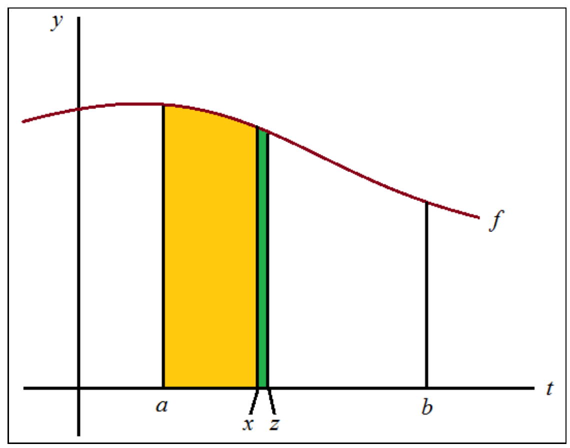


Figure 2.

First consider $A(z) - A(x)$; this corresponds to the slim rectangle-like figure in Figure 2. Notice that subtracting $A(x)$ from $A(z)$ leaves almost nothing since x and z are very close together. Next, we consider the expression $\dfrac{A(z)-A(x)}{z-x}$. This has the structure of $\dfrac{\text{area}}{\text{width}}$ which is a "height" of sorts. Notice that $\dfrac{A(z)-A(x)}{z-x} = \dfrac{\text{area}}{\text{width}} \approx f(z)$ (study **Figure 2** carefully). Finally, let $z \to x$ (look at **Figure 2** again to make sense of this). We get $\lim_{z\to x} \dfrac{A(z)-A(x)}{z-x} = \lim_{z \to x} f(z)$ or $A'(x) = f(x)$. Since $A(x) = \int_a^x f(t)\,dt$, we often see $A'(x) = f(x)$ expressed as $\dfrac{d}{dx}\int_a^x f(t)\,dt = f(x)$.