MATH 166
Lesson 4.3b
Fundamental Theorem of Calculus II

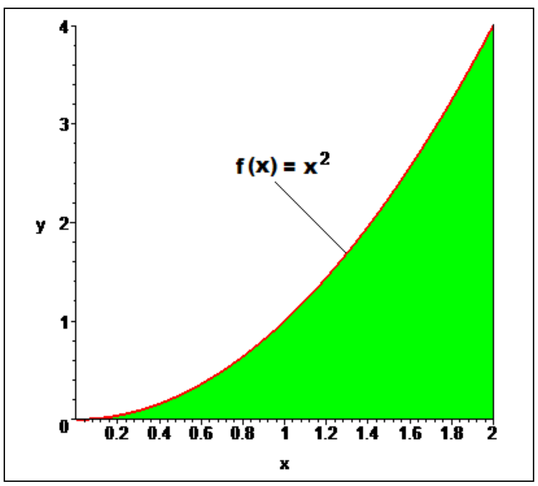
The Fundamental Theorem of Calculus (Part 2) is the main feature of this section; we'll get right to it:

> Fundamental Theorem of Calculus, Part 2: Let the function $f(x)$ be continuous on the interval $[a,b]$ and let $F(x)$ be an antiderivative of $f(x)$. Then
>
> $$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}.$$

Without hesitation, this is the **most important** theorem in Calculus. It gives us a very efficient way to calculate the area under the curve. First, find the antiderivative $F(x)$ from Lesson 3.9. Then subtract at the endpoints of the interval $[a,b]$. The answer is just $F(b)-F(a)$. Once this theorem is fully understood, there is little need to sketch rectangles, use sums, take limits, etc. Here is an example.

Example: Evaluate $\int_0^2 x^2 dx$.

Solution: According to FTC II, first find an antiderivative of x^2, evaluate it at the endpoints of $[0,2]$, and subtract. We know that an antiderivative of $f(x)=x^2$ is $F(x)=\frac{x^3}{3}$. Then $F(2)-F(0)=\frac{2^3}{3}-\frac{0^3}{3}=\frac{8}{3}$. Thus, $\int_0^2 x^2 dx = \frac{8}{3}$ units^2. Remember what was just calculated:



Note 1: Compare this with the methods seen in earlier lessons (e.g., using a finite sum to approximate the area, using the limit of a sum to find the exact area, using a geometric formula if such a formula applies, etc.). It's pretty clear that FTC II supplants all of these methods.

Note 2: The notation often used when applying FTC II is as follows:

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a).$$

The previous example can be expressed in the following manner:

$$\begin{aligned}\int_0^2 x^2\,dx &= \frac{x^3}{3}\Big|_0^2 \\ &= \frac{2^3}{3} - \frac{0^3}{3} \\ &= \frac{8}{3}.\end{aligned}$$

Note 3: It is important that you know your antiderivatives (Lesson 3.9) or at least how to find them. Without this skill, FTC II won't take you very far.

Note 4: What happened to the "$+C$" from antidifferentiation? Notice that we haven't been writing it down! The fact is, **we don't need to write it down when finding areas under the curve**. Look what happens if we decide to include it:

$$\begin{aligned}\int_a^b f(x)\,dx &= F(x) + C\Big|_a^b \\ &= \left[F(b) + C\right] - \left[F(a) + C\right] \\ &= F(b) - F(a).\end{aligned}$$

This is why we haven't bothered to use it at all in this section.

Note 5: In class, we will practice using the FTC II and we will also look at how FTC I & FTC II are both similar and different. Look back at each statement and see if you can detect similarities and differences:

FTC I: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$	FTC II: $\int_a^b f(x)\,dx = F(x)\Big\|_a^b = F(b) - F(a)$