

Fundamental Theorem, Part II

$$(A) \int_1^4 2\sqrt{x} \, dx$$

$$= \left. \frac{2x^{3/2}}{3/2} \right|_1^4 = \frac{4}{3} x^{3/2} \Big|_1^4$$

$$= \left[\frac{4}{3} (4)^{3/2} \right] - \left[\frac{4}{3} (1)^{3/2} \right]$$

$$= \frac{4}{3} (8) - \frac{4}{3} (1)$$

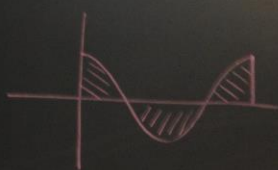
$$= \frac{32}{3} - \frac{4}{3} = \frac{28}{3}$$

$$(B) \int_0^{2\pi} \cos t \, dt$$

$$= \sin t \Big|_0^{2\pi}$$

$$= \sin 2\pi - \sin 0$$

$$= 0 - 0$$

$$= 0$$



Notice that a picture (using the net area concept) supports why this “area” is zero.

$$(C) \int_0^{\pi/6} (1 - \sin \theta) \, d\theta = \theta + \cos \theta \Big|_0^{\pi/6}$$

$$= \left[\frac{\pi}{6} + \cos\left(\frac{\pi}{6}\right) \right] - \left[0 + \cos(0) \right]$$

$$= \left(\frac{\pi}{6} + \cos\left(\frac{\pi}{6}\right) \right) - 1$$

$$= 1.3896$$

$$\frac{\pi}{6} + \frac{\sqrt{3}}{2} - 1$$


The answer is around 0.3896 (exact answer is

also displayed: $\frac{\pi}{6} + \frac{\sqrt{3}}{2} - 1$)

$$(D) \int_2^3 \frac{\sin x}{x} \, dx = 0$$

Endpoints are the same

$$\int_2^3 \frac{\sin x}{x} \, dx = F(3) - F(2)$$

$$\frac{\pi}{6} + \frac{\sqrt{3}}{2} - 1$$

Notice: $\frac{\sin x}{x}$ has no obvious antiderivative!

$$\begin{aligned}
 (E) \int_{-4}^1 (x-4)(1-x) dx \\
 \int_{-4}^1 (-x^2 + 5x - 4) dx \\
 \left[-\frac{x^3}{3} + \frac{5x^2}{2} - 4x \right]_{-4}^1 \\
 \left(-\frac{1^3}{3} + \frac{5(1)^2}{2} - 4(1) \right) - \left(-\frac{(-4)^3}{3} + \frac{5(-4)^2}{2} - 4(-4) \right) \\
 \underline{\underline{4.5}}
 \end{aligned}$$

F(2)

$$\begin{aligned}
 (F) \int_{-4}^1 |x| dx \\
 = \int_{-4}^0 (-x) dx + \int_0^1 x dx \\
 = \left[-\frac{1}{2}x^2 \right]_{-4}^0 + \left[\frac{1}{2}x^2 \right]_0^1 \\
 = \left[0 - \frac{1}{2}(-4)^2 \right] + \left[\frac{1}{2} - 0 \right] \\
 = 8 + \frac{1}{2} \\
 = \frac{17}{2}
 \end{aligned}$$

Two ways to attack this one: (1) "Breaking" $|x|$ into its two simple functions $y = x$ and $y = -x$ and then integrating, or (2) drawing a picture and observing that we are finding the areas of rectangles.

When does FTC II fail us???

(1) It is not very helpful if we cannot find an antiderivative. That is, what good is

$$\int_a^b F'(x) dx = F(b) - F(a) \text{ if you cannot find } F?$$

(2) Infinite discontinuities are also problematic (see below):

