

Indefinite Integrals

Level I:

a. $\int x^{4/3} dx = \frac{3}{7} x^{7/3} + C$

b. $\int (t^2 + 4t + 5) dt = \frac{t^3}{3} + 2t^2 + 5t + C$

c. $\int \sec^2 \theta d\theta = \tan \theta + C$

Level II:

a. $\int (3x+5)^2 dx$
 $\int (9x^2 + 30x + 25) dx$
 $3x^3 + 15x^2 + 25x + C$

b. $\int \left(\frac{x^3 + 4x}{x} \right) dx$
 $\int (x^2 + 4) dx$
 $\frac{1}{3} x^3 + 4x + C$

c. $\int \sin(3x) dx$
 $-\frac{1}{3} \cos(3x) + C$

Level III:

a) $\int \tan^2 x \, dx = \int \frac{\sin^2 x}{\cos^2 x} \, dx$
 $\int \sin^2 x \cdot \frac{1}{\cos^2 x} \, dx$
 $\sin^2 x = 1 - \cos^2 x$
 $\int (1 - \cos^2 x) \cdot \left(\frac{1}{\cos^2 x}\right) dx$
 $\int \frac{1}{\cos^2 x} - 1 \, dx$
 $\int \sec^2 x - 1 \, dx$
 $\tan x - x + C$

The above board makes use of the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$ while this board (see below)

a) $\int \tan^2 x \, dx$
 $\int (\sec^2 x) - 1 \, dx$
 $\int \sec^2 x \, dx - \int 1 \, dx =$
 $\tan(x) - x + C$

makes direct use of $\tan^2 \theta + 1 = \sec^2 \theta$. Note, however, that $\tan^2 \theta + 1 = \sec^2 \theta$ can be derived from $\sin^2 \theta + \cos^2 \theta = 1$.