a) $\int \tan^2 x \, dx$

$\int \sec^2(x) - 1 \, dx$

$\int \sec^2(x)\,dx - \int 1 \, dx =$

$\tan(x) - x + c$

b) $\int \frac{1}{1-\cos x} \, dx = \int -\frac{1}{\cos x - 1} \, dx = \int -\frac{\sec^2(\frac{x}{2})}{2\tan^2(\frac{x}{2})}$

$* \; u = \tan(\frac{x}{2})$ & $\frac{du}{dx} = \frac{\sec^2(\frac{x}{2})}{2}$

$= -\int \frac{1}{u^2} \, du = \frac{1}{u} = \frac{1}{\tan\frac{x}{2}}$

$-\int \frac{1}{\cos x - 1} \, dx = -\frac{1}{\tan(\frac{x}{2})} + c = -\cot(\frac{x}{2}) + c$

Wolfram|Alpha Step-by-step Solution

Indefinite integral:

Take the integral:

$$\int \frac{1}{1-\cos(x)} \, dx$$

For the integrand $\frac{1}{1-\cos(x)}$, substitute $u = \tan\left(\frac{x}{2}\right)$ and $du = \frac{1}{2} \, dx \sec^2\left(\frac{x}{2}\right)$. Then transform the integrand using the substitutions $\sin(x) = \frac{2u}{u^2+1}$, $\cos(x) = \frac{1-u^2}{u^2+1}$ and $dx = \frac{2\,du}{u^2+1}$:

$$= \int \frac{2}{(u^2+1)\left(1 - \frac{1-u^2}{u^2+1}\right)} \, du$$

Simplify the integrand $\frac{2}{(u^2+1)\left(1 - \frac{1-u^2}{u^2+1}\right)}$ to get $\frac{1}{u^2}$:

$$= \int \frac{1}{u^2} \, du$$

The integral of $\frac{1}{u^2}$ is $-\frac{1}{u}$:

$$= -\frac{1}{u} + \text{constant}$$

Substitute back for $u = \tan\left(\frac{x}{2}\right)$:

Answer:

$$= -\cot\left(\frac{x}{2}\right) + \text{constant}$$