

Substitution

$$\begin{aligned} (a) \int \sin^3 x \cos x dx \\ &= \int u^3 du \quad \begin{array}{l} u = \sin x \\ du = \cos x dx \end{array} \\ &= \frac{1}{4} u^4 + C \\ &= \frac{1}{4} \sin^4 x + C \end{aligned}$$

$$\begin{aligned} \int \sec(2z) \tan(2z) dz \\ u = 2z \\ \frac{du}{dz} = 2 \\ \frac{du}{2} = dz \\ \int \sec u \tan u \frac{du}{2} \\ &= \frac{1}{2} \int \sec u \tan u du \\ &= \frac{1}{2} \sec u + C \\ &= \frac{1}{2} \sec(2z) + C \end{aligned}$$

$$\begin{aligned} \int \sec^2 \theta \tan^5 \theta d\theta \\ u = \tan \theta \quad du = \sec^2 \theta d\theta \\ \int u^5 du \\ \frac{1}{6} u^6 + C \\ \frac{1}{6} \tan^6 \theta + C \end{aligned}$$

$$\begin{aligned} (e) \int \frac{\cos(\frac{1}{\theta})}{\theta^2} d\theta \quad \begin{array}{l} \text{let } u = \frac{1}{\theta} = \theta^{-1} \\ du = -\theta^{-2} d\theta \\ du = -\frac{1}{\theta^2} d\theta \end{array} \\ &= \int -\cos(u) du \\ &= -\int \cos(u) du \leftarrow * \\ &= -\sin(u) + C \\ &= -\sin\left(\frac{1}{\theta}\right) + C \end{aligned}$$

$$\begin{aligned}
 & \int (3x+4)^{10} dx \\
 & u = 3x+4 \\
 & \frac{du}{dx} = 3 \\
 & \int \frac{u^{10}}{3} du = \frac{1}{3} \left(\frac{u^{11}}{11} \right) \\
 & = \boxed{\frac{1}{33} (3x+4)^{11} + C}
 \end{aligned}$$

$$\begin{aligned}
 & f) \int x \sqrt{9-x^2} \\
 & u = 9-x^2 \\
 & du = -2x dx \\
 & \frac{du}{-2} = x dx \\
 & \int \sqrt{u} \cdot \frac{du}{-2} \\
 & -\frac{1}{2} \int u^{1/2} \\
 & -\frac{1}{2} \left(\frac{2u^{3/2}}{3} + C \right) \\
 & -\frac{1}{3} u^{3/2} + C \\
 & \boxed{-\frac{1}{3} (9-x^2)^{3/2} + C}
 \end{aligned}$$

$$\begin{aligned}
 & d) \int v^5 \sqrt{1+v^2} dx \\
 & \text{let } u = 1+v^2 \quad du = 2v dv \\
 & \frac{1}{2} du = v dv \\
 & v = u-1 \\
 & \int v^4 \cdot v \sqrt{1+v^2} dv = \int (u-1) (u-1)^{1/2} \frac{1}{2} du \quad v^4 = (u-1)^2 \\
 & = \frac{1}{2} \int (u^2 - 2u + 1) (u^{1/2}) du \\
 & = \frac{1}{2} \int (u^{5/2} - 2u^{3/2} + u^{1/2}) du \\
 & = \frac{1}{2} \left[\frac{2}{7} u^{7/2} - 2 \cdot \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right] \\
 & = \left[\frac{1}{7} u^{7/2} - \frac{2}{5} u^{5/2} + \frac{1}{3} u^{3/2} \right] \\
 & = \frac{1}{7} (1+v^2)^{7/2} - \frac{2}{5} (1+v^2)^{5/2} + \frac{1}{3} (1+v^2)^{3/2} + C
 \end{aligned}$$