

Substitution (Definite Integrals)

Converting the problem entirely to the u variable (including converting the limits of integration):

$$\int_0^1 \sqrt{1+6t} \, dt = \int_1^7 \sqrt{u} \frac{du}{6} = \frac{1}{6} \int_1^7 u^{1/2} \, du$$

$$\left. \begin{array}{l} u = 1+6t \\ \frac{du}{dt} = 6 \\ \frac{du}{6} = dt \end{array} \right\} \begin{array}{l} t=0 \rightarrow 1 \\ u=1 \rightarrow 7 \end{array}$$

$$= \frac{1}{6} \left. \frac{u^{3/2}}{3/2} \right|_1^7 = \frac{1}{9} \left. u^{3/2} \right|_1^7 = \frac{1}{9} (7^{3/2} - 1^{3/2}) \approx 1.947$$

The same problem but done with the t variable:

$$\int_0^1 \sqrt{1+6t} \, dt$$

$$= \frac{1}{6} \int_0^1 6\sqrt{1+6t} \, dt$$

$$= \frac{1}{6} \left. \left(\frac{(1+6t)^{3/2}}{3/2} \right) \right|_0^1 = \frac{1}{9} (1+6t)^{3/2} \Big|_0^1 = \frac{1}{9} (7^{3/2} - 1^{3/2}) = 1.945$$

*** The key to seeing the equivalence of the two methods is captured in the two expressions circled in red. Notice that the expressions *look* different but they evaluate to the same number. ***