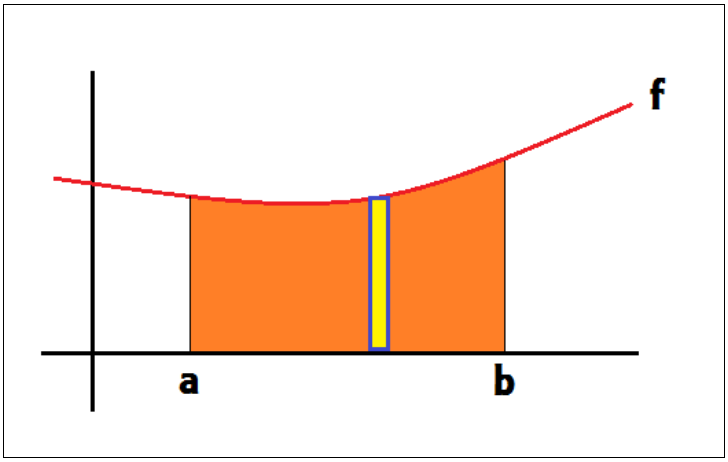
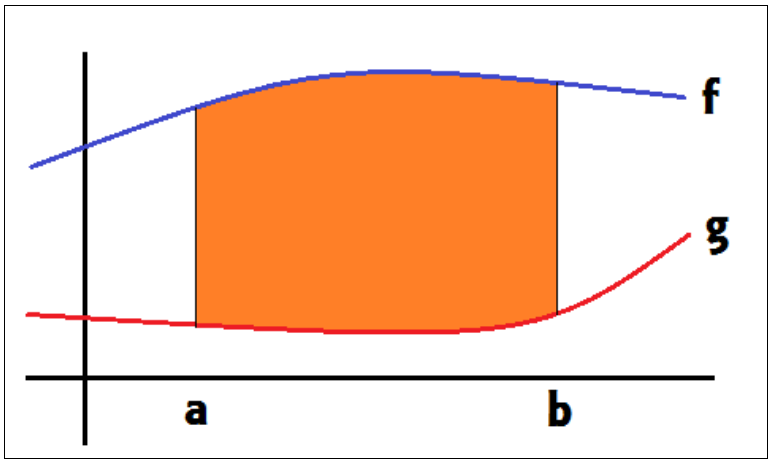
MATH 166
Lesson 5.1
Area Between Curves

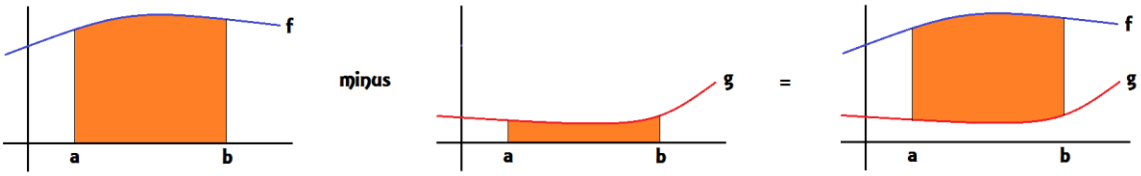
Recall from the previous chapter the idea of "area under the curve:"



The shaded area above (regardless of the irregularity of its shape) can be calculated exactly by $\int_a^b f(x)\,dx$. This obviously includes some assumptions (e.g., f is continuous on the interval $[a,b]$, f stays above the *x*-axis on $[a,b]$, etc.). In this lesson, we generalize this result to the area **between** two curves. For example, look at this picture:



This area can be quickly found by applying what we already know. We can find the area under f and then subtract the area under g; this would give the area bounded by (or between) the two curves. This can be visualized in the following way:



In symbols, we have

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b \left[f(x) - g(x) \right] dx \quad \leftarrow \text{Bounded Area}$$

This is commonly known as the "Top Curve minus the Bottom Curve" idea. Just be careful if the top curve/bottom curve switch roles at some point in the interval $[a,b]$.