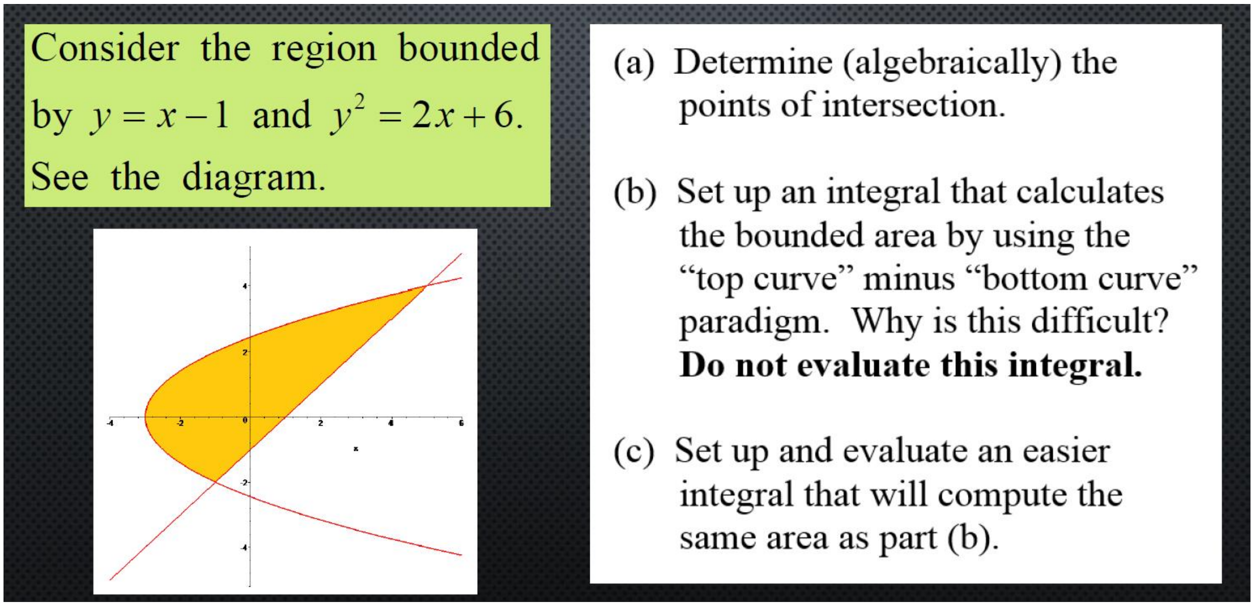
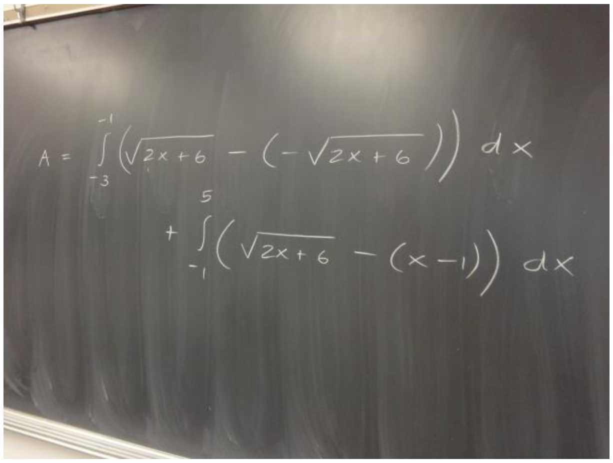
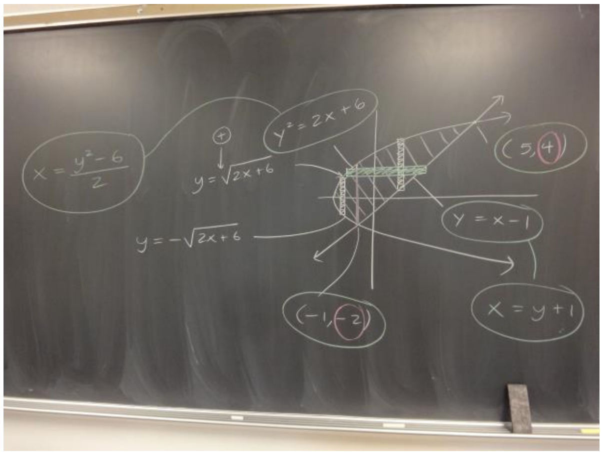
Area between two curves



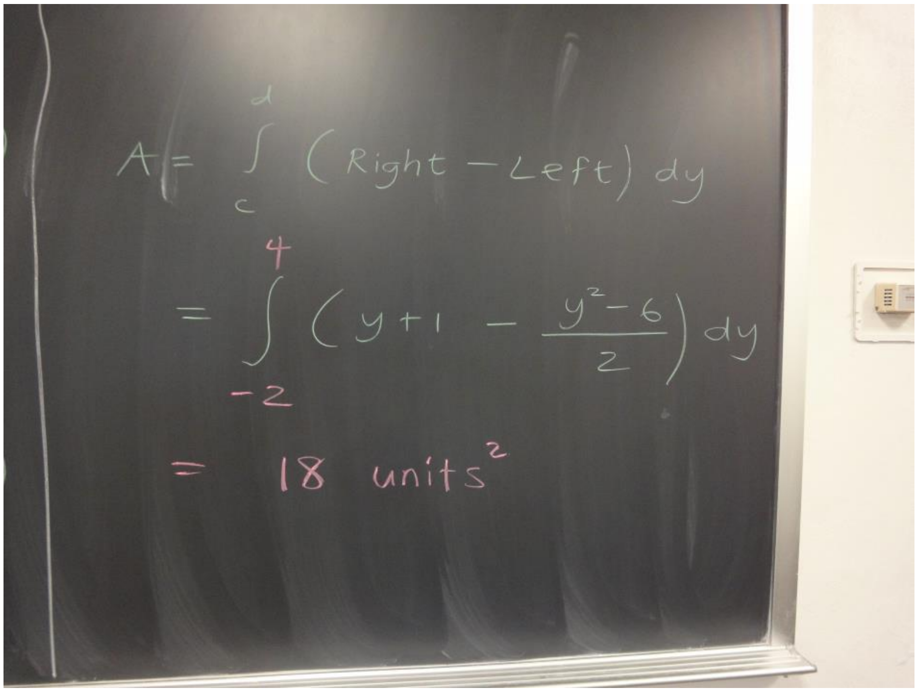
(a) Perhaps the simplest way to find the points of intersection is by setting y^2 equal to each other: $(x-1)^2 = 2x+6$ leads to $x^2 - 2x + 1 = 2x + 6$ or $x^2 - 4x - 5 = 0$ or $(x-5)(x+1) = 0$ leading to $x = 5$ and $x = -1$. So the points of intersection are $(5,4)$ and $(-1,-2)$.

(b) The "Top minus Bottom" model requires the use of **<u>two</u>** integrals because the bottom curve switches from the parabola $y^2 = 2x + 6$ to the line $y = x - 1$ at $x = -1$:





(c) An easier approach to this problem is to use *horizontal* rectangles instead of vertical ones. With this idea, we use the right curve (instead of the top curve) and the left curve (instead of the bottom curve). Moreover, everything needs to be done with respect to *y* (instead of *x*). The right curve is $x = y + 1$ and the left curve is $x = \frac{y^2 - 6}{2}$ and the integration limits vary from $y = -2$ to $y = 4$. So the integral is:



In this case, method (c) is better for two reasons—(1) it requires only a single integral, and (2) the integral is a bit easier than the one seen in part (b).