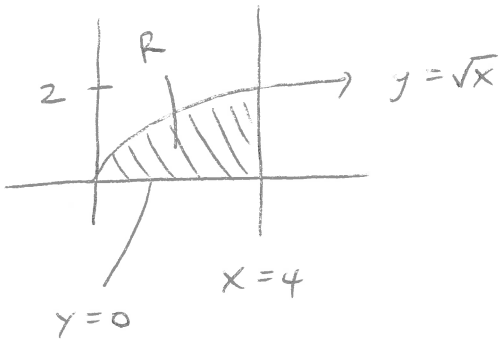
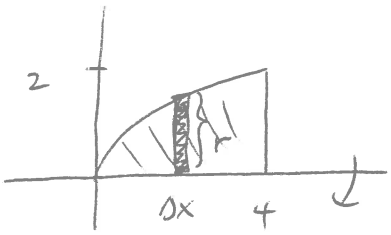
Method of Discs (Volume)



(A) x-axis

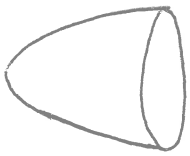


$$Vol = \pi \int_0^4 (\sqrt{x})^2 \, dx$$

$$= \pi \int_0^4 x \, dx$$

$$= 8\pi \text{ units}^3$$

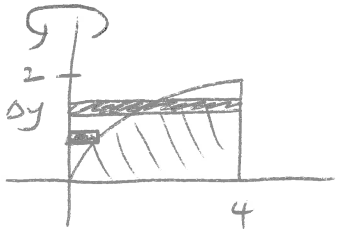
3D:



(B) y-axis

$y = \sqrt{x}$

$y^2 = x$



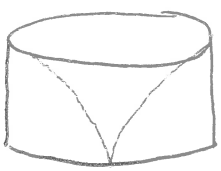
$R(y) = 4$

$r(y) = x$

$= y^2$

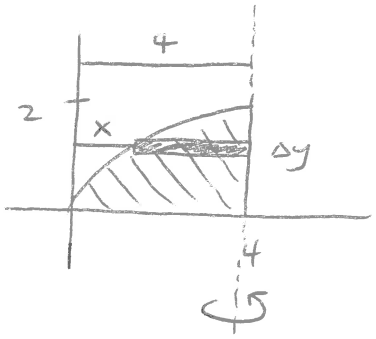
$$Vol = \pi \int_0^2 4^2 \, dy - \pi \int_0^2 (y^2)^2 \, dy$$

3D:



$$Vol = \pi \int_0^2 (16 - y^4) \, dy$$

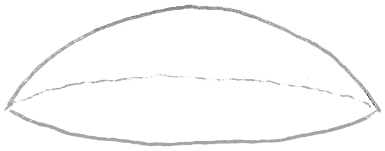
(c) line $x = 4$



$$r(y) = 4 - x$$
$$= 4 - y^2$$

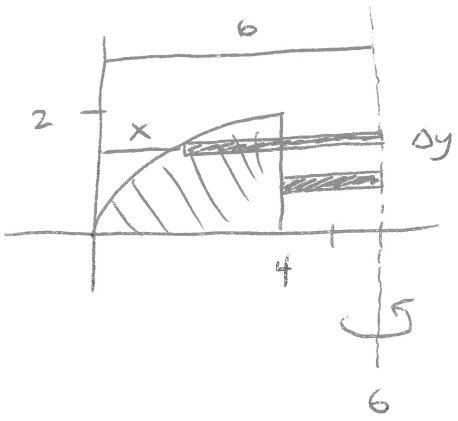
$$\text{Vol} = \pi \int_0^2 (4 - y^2)^2 \, dy$$

3D:



** Note the subtle differences between (B) & (C). Both use "4" and "y^2" but the resulting volume integrals are quite different!

(D) line $x = 6$



$$R(y) = 6 - x$$
$$= 6 - y^2$$

$$r(y) = 6 - 4$$
$$= 2$$

$$\text{Vol} = \pi \int_0^2 (6 - y^2)^2 \, dy - \pi \int_0^2 2^2 \, dy$$

$$= \pi \int_0^2 \left[(6 - y^2)^2 - 4 \right] dy$$

3D: