Properties of Matrix Operations

Section 2.2

Theorem

Let A, B, and C be matrices of the same size and let c and d be scalars. Then

(a) $A+B=B+A$

(b) $A+O=A$

(c) $(A+B)+C=A+(B+C)$

(d) $c(A+B)=cA+cB$

(e) $(c+d)A=cA+dA$

(f) $c(dA)=(cd)A$

Theorem

Let A, B, and C have sizes so that the indicated products and sums are defined. Then

(a) $A(BC) = (AB)C$

(b) $A(B+C) = AB + AC$

(c) $(B+C)A = BA + CA$

(d) $k(AB) = (kA)B = A(kB)$, k a scalar

(e) $I_m A = A = AI_n$ (note: A is $m \times n$)

Important!

1. If $AB = AC$, it is **not necessarily true** that $B = C$!!!
2. If $AB = O$, it need **not** be true that $A = O$ or $B = O$!!!

Example

Find the transpose of each matrix.

$$A = \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 2 & 4 \\ 1 & -1 \end{bmatrix}$$

Theorem

Let A and B be defined so that the sums and products are defined.

(a) $\left(A^T\right)^T = A$

(b) $\left(A+B\right)^T = A^T + B^T$

(c) For $c \in R$, $\left(cA\right)^T = cA^T$

(d) $\left(AB\right)^T = B^T A^T$

Example

Illustrate, for $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and

$B = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$ that $(AB)^T = B^T A^T$.