Problem

$$\text{Show that } A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ -2 & 3 & -2 \end{bmatrix}$$

is **not** invertible.

Elementary Matrices

Section 2.4

Definition

An **elementary matrix** is one that is obtained by performing a single ERO on an identity matrix.

Examples:

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Theorem

If E is an elementary matrix, then E^{-1} exists and it is also an elementary matrix.

Question

What elementary matrices would need to be multiplied (order is important!!) to produce $A = \begin{bmatrix} -1 & 2 \\ 5 & 0 \end{bmatrix}$?

Theorem (Equivalent Conditions)

Let A be $n \times n$. The following statements are **equivalent**.

(1) A is invertible.

(2) $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b} \in \mathbb{R}^n$.

(3) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.

(4) A is row equivalent to I_n.

(5) A can be expressed as the product of elementary matrices.