THEOREM

On the dimension of the solution space of $A\mathbf{x}=\mathbf{b}$ with A being $m\times n$:

$$\boxed{\operatorname{rank}(A)+\operatorname{nullity}(A)=n}$$

The **Invertible Matrix Theorem** just keeps on growing…

Let A be $n \times n$. The following statements are **equivalent**.

(1) A is invertible.

(2) $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b} \in \mathbb{R}^n$.

(3) $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.

(4) A is row equivalent to I_n.

(5) A can be expressed as the product of elementary matrices.

(6) $\det A \neq 0$

(7) The columns of A are linearly independent.

(8) The columns of A span $\mathbb{R}^n$.

(9) The columns of A form a basis for $\mathbb{R}^n$.

(10) $\operatorname{col}(A) = \mathbb{R}^n$

(11) $\dim(\operatorname{col}(A)) = n$

(12) $\operatorname{rank}(A) = n$

(13) $\operatorname{nullity}(A) = 0$

$\vdots$

THEOREM

If $\mathbf{x}_\mathbf{p}$ is a particular solution of the nonhomogeneous system $A\mathbf{x} = \mathbf{b}$, then every solution of this system can be written as

$$\boxed{\mathbf{x} = \mathbf{x}_\mathbf{p} + \mathbf{x}_\mathbf{h}}$$

where $\mathbf{x}_\mathbf{h}$ is a solution to the corresponding homogeneous system $A\mathbf{x} = \mathbf{0}$.