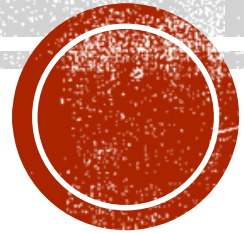
PROOF PRACTICE WITH DETERMINANTS

(a) Prove that if $|A| = |B| \neq 0$ and A and B are the same size, then there exists a matrix C such that $|C| = 1$ and $AC = B$.

(b) A square matrix A is called **nilpotent** if there exists a positive integer k such that $A^k = O$ (the zero matrix). Prove that if A is nilpotent, then $\det A = 0$.

VECTORS IN $\mathbb{R}^n$

Section 4.1



EXAMPLES OF COLUMN VECTORS

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \in \mathbb{R}^2, \quad \mathbf{b} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} -3 \\ 0 \\ 5 \end{bmatrix} \in \mathbb{R}^3,$$

$$\mathbf{d} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{e} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \mathbf{0}$$

EXAMPLE

Let $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Display the vectors

$\mathbf{u}$, $2\mathbf{u}$, and $-\frac{1}{2}\mathbf{u}$ graphically.