EXAMPLE

The set of all polynomials of degree 2 or less (this is typically denoted by P_2)

EXAMPLE

The set of all continuous functions on the real line – typically denoted by $C(-\infty, \infty)$

THEOREM

(Properties of Scalar Multiplication)

For $\mathbf{u} \in V$ and any scalar c,

1. $0\mathbf{u} = \mathbf{0}$
2. $c\mathbf{0} = \mathbf{0}$
3. If $c\mathbf{u} = \mathbf{0}$, then $c = 0$ or $\mathbf{u} = \mathbf{0}$.
4. $(-1)\mathbf{u} = -\mathbf{u}$

NONEXAMPLE

The set of integers along with the standard operations

NONEXAMPLE

The set of second degree polynomials with the standard operations

NONEXAMPLE

Let $V = \mathbb{R}^2$ with the standard vector addition and the following **nonstandard** definition of scalar multiplication:

$$c\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} cx_1 \\ 0 \end{bmatrix}$$

Show that V is **not** a vector space.

VECTOR SPACES? YES OR NO?

(a) The set of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ c & 0 \end{bmatrix}$.

(b) The set of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ c & 1 \end{bmatrix}$.

(c) The set of all 2×2 singular (noninvertible) matrices.