LINEAR TRANSFORMATIONS

Section 6.1

PROPERTIES OF LINEAR TRANSFORMATIONS

Let $T: V \to W$ be linear with $\mathbf{u}, \mathbf{v} \in V$. Then

(1) $T(\mathbf{0}) = \mathbf{0}$

(2) $T(-\mathbf{v}) = -T(\mathbf{v})$

(3) $T(\mathbf{u} - \mathbf{v}) = T(\mathbf{u}) - T(\mathbf{v})$

(4) $T(c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n)$

$$= c_1T(\mathbf{v}_1) + c_2T(\mathbf{v}_2) + \cdots + c_nT(\mathbf{v}_n)$$

EXAMPLE

Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be linear with

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}2\\-1\\4\end{bmatrix},\quad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\5\\-2\end{bmatrix},\ \text{and}$$

$$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}0\\3\\1\end{bmatrix}.\ \text{What is } T\left(\begin{bmatrix}2\\3\\-2\end{bmatrix}\right)?$$