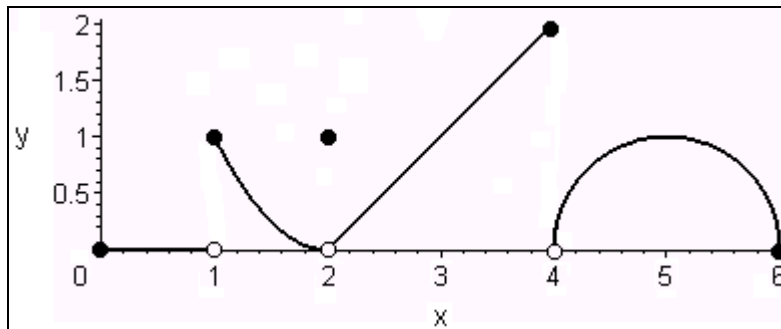


MTH 150
Sample Exam 1

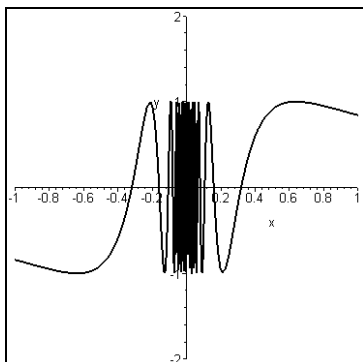
Name: _____

DIRECTIONS: This is a closed book, closed notes exam. No electronic devices are allowed (this means calculators, computers, cell phones, pagers, etc.). Be neat and show all work to receive full credit. Correct answers without the supporting evidence to back it up receive only partial credit. Good luck.

1. (16 points) Consider the graph of $y = f(x)$ given below. Fill in the blanks appropriately. If the limit does not exist, then write DNE (does not exist).



- (a) $\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}}$ (b) $\lim_{x \rightarrow 2} f(x) = \underline{\hspace{2cm}}$
- (c) $\lim_{x \rightarrow 1^+} f(x) = \underline{\hspace{2cm}}$ (d) $f(2) = \underline{\hspace{2cm}}$
- (e) $\lim_{x \rightarrow 1} f(x) = \underline{\hspace{2cm}}$ (f) $\lim_{x \rightarrow 4^-} f(x) = \underline{\hspace{2cm}}$
- (g) $f(4) = \underline{\hspace{2cm}}$ (h) $\lim_{x \rightarrow 6^-} f(x) = \underline{\hspace{2cm}}$
2. (9 points) Give a detailed explanation as to why $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$ does not exist. You may use the graph of $f(x) = \sin\left(\frac{1}{x}\right)$ in your explanation (shown below).



For exercises 3-6, evaluate the limit analytically. In other words, use some algebra or a well-known Calculus result to arrive at your answer.

3. (7 points) $\lim_{r \rightarrow -2} (r^3 - 2r^2 + 5r - 1)$

4. (7 points) $\lim_{\theta \rightarrow 0} \frac{\sin(3\theta)}{4\theta}$

5. (7 points) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

6. (7 points) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$

7. **(8 points)** Explain two different ways that a function can fail to be continuous at a point. Draw a picture to supplement each explanation.

(i) **Explanation:**

Picture:

(ii) **Explanation:**

Picture:

8. **(9 points)** Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - \sqrt{x^2 - x})$ by rationalizing.

9. Consider the function $u(x) = \frac{x \sin x}{2 - 2 \cos x}$.

(a) **(6 points)** It can be shown that the inequalities

$$1 - \frac{x^2}{6} < \frac{x \sin x}{2 - 2 \cos x} < 1$$

hold for x close to zero. What, if anything, does this tell you about $\lim_{x \rightarrow 0} u(x)$?

Give reasons for your answer.

(b) **(3 points)** What happens if you attempt to compute $u(0)$? Does this contradict the result in part (a)? Why or why not?

For problems 10-12 answer TRUE or FALSE. No explanation is necessary. Each question is worth **1 point**.

10. _____ If $\lim_{x \rightarrow c^+} f(x) = L$ and $\lim_{x \rightarrow c^-} f(x) = L$ then $f(x)$ is continuous at $x = c$.

11. _____ If $\lim_{x \rightarrow c} f(x)$ does not exist, then the graph of $f(x)$ has a vertical asymptote at $x = c$.

12. _____ If $f(x) > g(x)$ for all $x \neq 5$, then $\lim_{x \rightarrow 5} f(x) > \lim_{x \rightarrow 5} g(x)$.

BONUS (Optional 5 points)

Note: Only proofs written correctly will earn the 5 points.

Use the $\varepsilon - \delta$ definition to prove that $\lim_{x \rightarrow 3} (4x - 1) = 11$. In other words, show that, for every $\varepsilon > 0$, there is a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

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DIRECTIONS: Calculators are permitted on this part of the exam. However, answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

13. (6 points) Given the function $f(x) = x^3 - 8x + 10$, show that there is a value c for which $f(c) = \pi$. What is the name of the theorem that guarantees this?

14. (12 points) Evaluate the following limits (no explanation is necessary).

(a) $\lim_{x \rightarrow 0} (1+x)^{1/x} =$

(b) $\lim_{x \rightarrow \pi/2^+} \tan x =$

(c) $\lim_{x \rightarrow \infty} \frac{x^3 - 4x^2}{1 - 5x^3} =$

(d) $\lim_{x \rightarrow 5^-} \frac{x-3}{x-5} =$