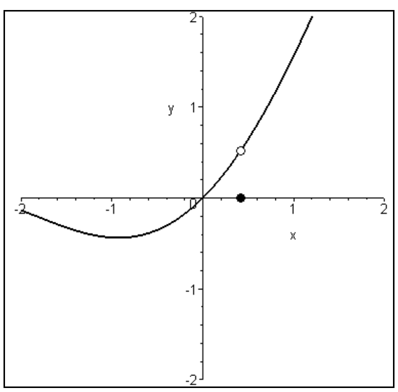
MTH 150
Sample Exam 1
Key

1. (a) 0 (b) 0 (c) 1 (d) 1
   (e) does not exist (f) 2 (g) 2
   (h) 0

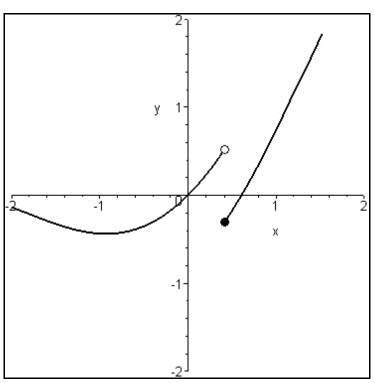
2. Study the graph of $y = \sin\left(\frac{1}{x}\right)$ as *x* gets closer and closer to zero from either direction. Because the function is oscillating wildly, $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist. In other words, $y = \sin\left(\frac{1}{x}\right)$ does not approach any one number as $x \to 0$.

3. -27
4. 3/4
5. 1/4
6. 5

7. The graph could have a "hole" or removable discontinuity. See the picture below.



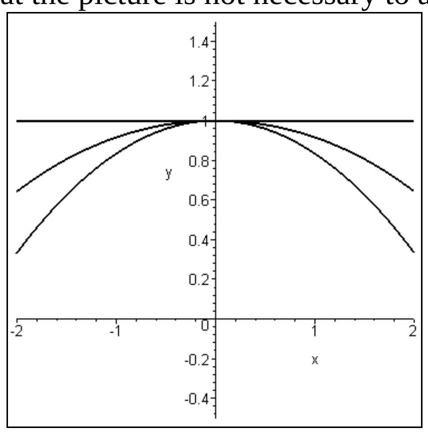
On the other hand, the limit may not exist at the point in question. See the picture.



8. Proceed as follows:

$$\begin{aligned}
\lim_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right) &= \lim_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)\cdot\frac{\sqrt{x^2+x}+\sqrt{x^2-x}}{\sqrt{x^2+x}+\sqrt{x^2-x}} \\
&= \lim_{x\to\infty}\frac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}} \\
&= \lim_{x\to\infty}\frac{2x}{x\sqrt{1+\frac{1}{x}}+x\sqrt{1-\frac{1}{x}}} \\
&= \lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{1}{x}}+\sqrt{1-\frac{1}{x}}} \quad \text{(now let } x\to\infty\text{)} \\
&= \frac{2}{\sqrt{1}+\sqrt{1}} \\
&= 1
\end{aligned}$$

9. (a) By the Squeeze Theorem, $\lim_{x\to 0} u(x)=1$, since, as $x\to 0$ both $1-\frac{x^2}{6}\to 1$ and $1\to 1$. That is, $u(x)$ is "squeezed" between $y=1-\frac{x^2}{6}$ and $y=1$. See the picture below (note that the picture is not necessary to answer this question).



(b) $u(0)$ is not defined (it results in a 0/0 form). This does not contradict the fact that $\lim_{x\to 0} u(x)=1$. Remember that a function can have a limit at a point yet be undefined there. There is a "hole" in the graph of $y=u(x)$ at the point $(0,1)$.

10. False
11. False
12. False

13. First note that any polynomial is continuous. Therefore, we can say that $f(x)$ is continuous on the closed interval $[0,1]$. Now check that $f(0)=10$ and $f(1)=3$. The Intermediate Value Theorem guarantees that the function $f(x)$ must assume every value between 3 and 10. Since $3<\pi<10$, there must be a value c in the interval $[0,1]$ for which $f(c)=\pi$.

14. (a) e (b) $-\infty$ (c) $-1/5$ (d) $-\infty$