MTH 150 **Name: _____**
Exam 1
February 6, 2007

DIRECTIONS: This is a <u>closed book</u>, <u>closed notes</u> exam. <u>No electronic devices</u> are allowed (this means calculators, computers, cell phones, pagers, etc.). Be neat and <u>show all work</u> to receive full credit. Correct answers without the supporting evidence to back it up receive only partial credit. Good luck.

For exercises 1-3, evaluate the limit analytically. In other words, use some algebra or a well-known Calculus result to arrive at your answer.

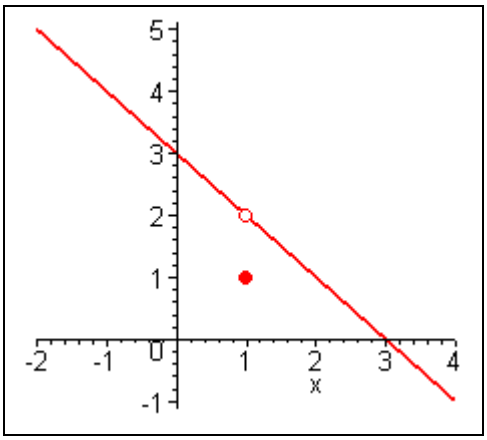
1. **(10 points)** $\lim_{x\to 3} \dfrac{x^2 - 4x}{e^x}$

2. **(10 points)** $\lim_{\Delta x\to 0} \dfrac{(x+\Delta x)^2 - x^2}{\Delta x}$

3. **(10 points)** $\lim_{h\to 3} \dfrac{\sqrt{h+1}-2}{h-3}$

Problems 4-8 are multiple choice; no work is necessary. Circle the letter of your choosing.

4. **(5 points)** Use the graph of $f(x)=\begin{cases}3-x, & x\neq 1\\ 1, & x=1\end{cases}$ below to find $\lim_{x\to 1} f(x)$, if it exists.



(a) 0
(b) 1
(c) 2
(d) 3
(e) does not exist

5. **(5 points)** State where the function $g(x)=\dfrac{x+2}{x^2-x-6}$ is continuous.

(a) $(-\infty,-2)\cup(-2,\infty)$
(b) $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$
(c) $(-\infty,3)\cup(3,\infty)$
(d) $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$
(e) $(-\infty,-3)\cup(-3,\infty)$

6. **(5 points)** Find $\lim_{x\to\infty}\dfrac{x^2-2x}{3x^2-5x+1}$, if it exists.

(a) 1/3
(b) 0
(c) 2/5
(d) 3
(e) does not exist

7. **(5 points)** Find $\lim_{\theta \to 0} \dfrac{\cos\theta \tan\theta}{\theta}$, if it exists.

(a) 0
(b) 1
(c) e
(d) $\pi / 2$
(e) does not exist

8. **(5 points)** Find $\lim_{t \to 0} (1+t)^{4/t}$, if it exists.

(a) $4e$
(b) e^4
(c) e
(d) 1
(e) does not exist

BONUS (Optional 5 points)
Note: Only proofs written <u>correctly</u> will earn the 5 points.

Prove that $\lim_{x \to 2} (2x+1) = 5$ via the $\varepsilon - \delta$ definition. That is, show that, for every $\varepsilon > 0$, there is a $\delta > 0$ such that, if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

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DIRECTIONS: Calculators are permitted on this part of the exam. However, answers based solely on calculator results are unacceptable. You must still <u>show all work</u> to receive full credit. Good luck.

9. **(9 points)** Consider the function $h(x)=\dfrac{|x-7|}{x-7}$. Does $\lim\limits_{x\to 7} h(x)$ exist? If so, find the limit. If not, explain why not.

10. **(9 points)** Prove that $f(x)=x^3+x-1$ has a zero somewhere in the interval $[0,1]$. What is the name of the theorem that guarantees this?

11. Consider the function $f(x)=\dfrac{x^2+7x+12}{x^2-9}$.

(a) **(6 points)** Find the value(s) of x at which $f(x)$ is discontinuous and label each discontinuity as removable or non-removable.

(b) **(6 points)** Write down an appropriate limit to show that $f(x)$ has a vertical asymptote. Give an equation for the VA.

Limit: _____ **VA: _____**

(c) **(6 points)** Use an appropriate limit to show that $f(x)$ has a horizontal asymptote. Give an equation for the HA.

Limit: _____ **HA: _____**

12. **(9 points)** Given a function $y=f(x)$, explain three different ways that $\lim\limits_{x\to c} f(x)$ can fail to exist. Feel free to provide a picture if you wish.

(a)

(b)

(c)