MTH 150
Exam 1
Solutions

Note: Some of these solutions contain many details; others do not.

1. $-3/e^3$

2. Use the FOIL method on the term $(x+\Delta x)^2$ and then simplify as usual:

$$\begin{aligned}\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-x^2}{\Delta x} &= \lim_{\Delta x\to 0}\frac{x^2+2x\Delta x+(\Delta x)^2-x^2}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{2x\Delta x+(\Delta x)^2}{\Delta x}\\ &= \lim_{\Delta x\to 0}(2x+\Delta x)\\ &= 2x.\end{aligned}$$

3. 1/4

4. C

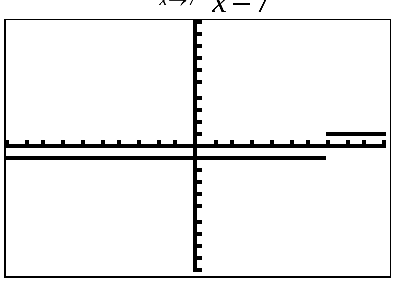
5. $g(x)=\dfrac{x+2}{x^2-x-6}=\dfrac{x+2}{(x+2)(x-3)}$. By studying this form, there are **two** discontinuities—one at $x=-2$ (a hole) and one at $x=3$ (a vertical asymptote). Thus, the function is continuous on $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$. The answer is B.

6. A

7. B

8. B

9. $\lim_{x\to 7}\dfrac{|x-7|}{x-7}$ does not exist because $\lim_{x\to 7^+}\dfrac{|x-7|}{x-7}=1$ yet $\lim_{x\to 7^-}\dfrac{|x-7|}{x-7}=-1$. Since the one-sided limits differ, $\lim_{x\to 7}\dfrac{|x-7|}{x-7}$ fails to exist. See the picture below.



10. $f(0) = -1$ while $f(1) = 1$. Since $f(x)$ is continuous, it must take on every value between -1 and 1. In other words, $f(c) = 0$ for some *c* value in $[0,1]$. The Intermediate Value Theorem guarantees this.

11. (a) Similar to problem 5 above, write $f(x) = \frac{x^2 + 7x + 12}{x^2 - 9} = \frac{(x+3)(x+4)}{(x+3)(x-3)}$. From this alone, it is clear that $f(x)$ has two discontinuities—one at $x = 3$ and one at $x = -3$. $x = 3$ is a nonremovable discontinuity (a vertical asymptote) while $x = -3$ is a removable discontinuity (a hole).
(b) $\lim_{x \to 3} f(x)$ fails to exist due to the function's unbounded behavior. The equation for the VA is $x = 3$.
(c) $\lim_{x \to \infty} f(x) = 1$ indicates that there is a horizontal asymptote at $y = 1$.

12. (a) A limit may fail to exist if the one-sided limits are different; the function may have a "jump" discontinuity.
(b) A limit may fail to exist if a function oscillates uncontrollably (i.e., the function does not approach any one fixed value).
(c) A limit may fail to exist because of unbounded behavior (a vertical asymptote).