

MTH 150
Sample Exam 2

Name: _____

DIRECTIONS: This is a closed book, closed notes exam. No electronic devices are allowed (this means calculators, computers, cell phones, pagers, etc.). Be neat and show all work to receive full credit. Correct answers without the supporting evidence to back it up receive only partial credit. Good luck.

1. (a) (3 points) Given a function $y = f(x)$, state the **definition** of the derivative.

(b) (4 points) Based on the definition from part (a), find $f'(x)$ for $f(x) = \sqrt{x+1}$.
You may *check* your answer by using any well-known shortcut (optional).

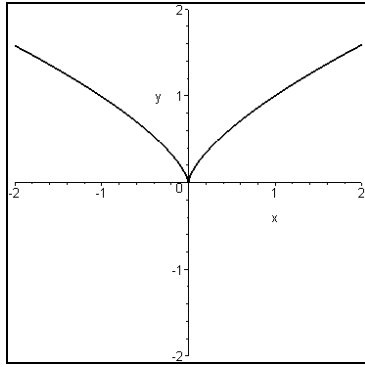
(c) (4 points) Explain, geometrically, what the derivative represents. Be clear in your explanation.

2. (7 points) Explain why the function $f(x) = x^{2/3}$ is not differentiable at the point $c = 0$. You should use at least one of the following in your argument:

(i) the alternative form of the derivative $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$

(ii) one-sided limits

(iii) the graph of $f(x) = x^{2/3}$:



3. (5 points) Explain why the Quotient Rule is not necessary to differentiate the function $f(x) = \frac{x^2 - x + 5}{x^3}$. You need not find $f'(x)$; just provide an explanation.

For problems 4-9, compute the derivative. Simplify answers to a reasonable point (no negative exponents, combine like terms, etc.).

4. (7 points) $h(x) = \frac{2}{\sqrt[3]{x}} - 5 \cos x + e^{7x}$

5. (7 points) $f(x) = \frac{\sec x}{x}$

6. (7 points) $g(\theta) = \cos^3 \theta + \cos \theta^3$

7. (7 points) $y = x\sqrt{1-x^2}$

8. (7 points) $a(t) = 3^t + \log_3 t$

9. (7 points) $h(x) = x^2 \arctan x$

10. Consider the relation $y^3 + y^2 - 5y - x^2 = -4$.

(a) **(7 points)** Find $\frac{dy}{dx}$.

(b) **(6 points)** Find the equation of the tangent line at the point $(2, 0)$.

11. **(6 points)** Given the function $s(t)$, indicating the position of an object, describe the meaning of the functions $s'(t)$ and $s''(t)$.

BONUS (Optional 10 points)

Show that the function $f(x) = \arcsin\left(\frac{x-2}{2}\right) - 2\arcsin\left(\frac{\sqrt{x}}{2}\right)$ is constant for $0 \leq x \leq 4$.

Hint: Show that the derivative of this function is zero.

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DIRECTIONS: Calculators are permitted on this part of the exam. However, answers based solely on calculator results are unacceptable. You must still show all work to receive full credit. Good luck.

12. (8 points) The edges of a cube are expanding at a rate of 5 centimeters per second. How fast is the surface area changing when each edge is 4.5 cm?

13. (8 points) Use Newton's method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ to approximate the zero of the function $f(x) = x^3 - 3x + 4$. Use $x_0 = -2$ as an initial guess and find x_1 , x_2 , and x_3 . Show the work for the computation of x_1 ; otherwise let the calculator do the work.