MTH 150
Sample Exam 2
Key

1. (a) $f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

   (b) Proceed as follows:

$$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0}\frac{\sqrt{x+\Delta x+1}-\sqrt{x+1}}{\Delta x} \\ &= \lim_{\Delta x\to 0}\frac{\sqrt{x+\Delta x+1}-\sqrt{x+1}}{\Delta x}\cdot\frac{\sqrt{x+\Delta x+1}+\sqrt{x+1}}{\sqrt{x+\Delta x+1}+\sqrt{x+1}} \\ &= \lim_{\Delta x\to 0}\frac{x+\Delta x+1-x-1}{\Delta x\left(\sqrt{x+\Delta x+1}+\sqrt{x+1}\right)} \\ &= \lim_{\Delta x\to 0}\frac{1}{\sqrt{x+\Delta x+1}+\sqrt{x+1}} \\ &= \frac{1}{2\sqrt{x+1}} \end{aligned}$$

   (c) It gives the slope of the tangent line to the graph of $y=f(x)$ at the point $(x,f(x))$.

2. There is a sharp change in direction on the graph at $x=0$. Because of this cusp, the function is not differentiable at the point $x=0$. You may also look at the one sided limits. We know that $f'(0)=\lim_{x\to 0}\frac{x^{2/3}-0}{x-0}=\lim_{x\to 0}\frac{x^{2/3}}{x}=\lim_{x\to 0}\frac{1}{x^{1/3}}$. As $x\to 0^+$, $\frac{1}{x^{1/3}}\to\infty$. On the other hand, as $x\to 0^-$, $\frac{1}{x^{1/3}}\to-\infty$. Since these limits are in disagreement, $f'(0)$ does not exist.

3. You may rewrite $f(x)$ as $f(x)=\frac{1}{x}-\frac{1}{x^2}+\frac{5}{x^3}=x^{-1}-x^{-2}+5x^{-3}$ and use the Power Rule multiple times.

4. $h'(x)=-\frac{2}{3x^{4/3}}+5\sin x+7e^{7x}$
5. $\frac{\sec x(x\tan x-1)}{x^2}$
6. $g'(\theta)=-3\cos^2\theta\sin\theta-3\theta^2\sin\theta^3$

7. $y' = \frac{1-2x^2}{\sqrt{1-x^2}}$

8. $a'(t) = 3^t(\ln 3) + \frac{1}{\ln 3}\frac{1}{t}$

9. $h'(x) = \frac{x^2}{1+x^2} + 2x\arctan x$

10. (a) $\frac{dy}{dx} = \frac{2x}{3y^2+2y-5}$

    (b) $y = -\frac{4}{5}x + \frac{8}{5}$

11. s' represents velocity and s'' represents acceleration.

12. The area of each face is $x \cdot x = x^2$. Since a cube has six faces, we can write the surface area as $S = 6x^2$. Then $\frac{dS}{dt} = 12x \cdot \frac{dx}{dt}$. As a result,

$$\begin{aligned}\frac{dS}{dt} &= 12(4.5\text{ cm})(5\text{ cm/sec})\\ &= 270\text{ cm}^2/\text{sec}\end{aligned}$$

13. $x_1 \approx -2.222222,\ x_2 \approx -2.196215,\ x_3 \approx -2.195823$