

MTH 150
Exam 2
Solutions

Note: Some of these solutions contain many details; others do not.

1. (a) $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

(b) $f'(x) = \frac{1}{2\sqrt{x}}$

(c) The derivative gives the slope of the tangent line.

2. $f'(x) = 4x^3 - 14x$

3. Use the Quotient Rule: $f'(x) = \frac{(1 - \sin x)\cos x - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2}$. The answer

simplifies to $f'(x) = \frac{2\cos x}{(1 - \sin x)^2}$.

4. $g'(x) = \frac{-x^2}{\sqrt{5-x^2}} + \sqrt{5-x^2}$

5. $\frac{dy}{dx} = 4^x \ln 4 + \frac{1}{x \cdot \ln 4}$

6. The slope of the tangent line is given by $f'(x) = 2x - 1$. When $x = -2$, this slope is $f'(-2) = 2(-2) - 1 = -5$. This immediately narrows your choices to either (b) or (d). To finish the problem, use the point-slope formula which gives $y - 6 = -5(x + 2)$. This simplifies to $y = -5x - 4$, or B.

7. D

8. Differentiate both sides with respect to x ; this gives $e^{xy} \cdot [xy' + y(1)] = 2 + 3y'$. Isolate all y' terms on one side and then solve for y' . The answer is C.

9. B

10. First, rewrite the function as $f(x) = [\cos(5x)]^3$. Now it becomes clear that this is a power function. The Power Rule (and two applications of the Chain Rule) gives

$$\begin{aligned}f'(x) &= 3[\cos(5x)]^2 \cdot (-\sin(5x)) \cdot 5 \\ &= -15\cos^2(5x)\sin(5x).\end{aligned}$$

Choose E.

11. A

12. Since the coin is dropped (not thrown), the initial velocity is 0. In other words, $v_0 = 0$. Also, the building is 1300 ft above the ground so the initial height is given. So $s_0 = 1300$. Using this information, the position function is $s(t) = -16t^2 + 1300$ and the velocity function is $v(t) = s'(t) = -32t$. The *average velocity* corresponds to the slope of the secant line so this can be found by computing

$$\frac{\text{distance traveled}}{\text{time elapsed}} = \frac{s(2) - s(1)}{2 - 1} = -48 \text{ ft/sec}.$$

The instantaneous velocity for part (c) can be found by computing $v(2)$.

13. -1 , -0.6 , -0.4649 , -0.4535

14. This is done for you in the textbook. See the worked-out examples in Section 3.7 (Related Rates).