MTH 150   Name: _____
Sample Exam 3

DIRECTIONS: This is a <u>closed book</u>, <u>closed notes</u> exam. <u>No electronic devices</u> are allowed (this means calculators, computers, cell phones, pagers, etc.). Be neat and show all work to receive full credit. Correct answers without the supporting evidence to back it up receive only partial credit. Good luck.

1. **(7 points)** Determine whether Rolle's Theorem can be applied to the function $f(x) = x^4 - 2x^2$ on the closed interval $[-2, 2]$. If Rolle's Theorem can be applied, find all values of c in $(-2, 2)$ such that $f'(c) = 0$. If Rolle's Theorem cannot be applied then explain why this is the case.

2. **(8 points)** You are taking a drive on the highway in your newly acquired Lamborghini. Although you want to stop to get a drink, you carelessly fly by the first available rest area. Thirty miles and 25 minutes later, you pull into the next available rest area. What was your average speed over the thirty mile trip? What does the Mean Value Theorem say in relation to this average speed?
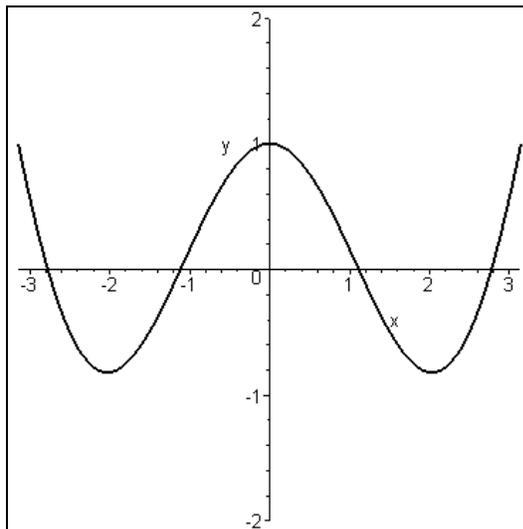
3. **(8 points)** Find the critical numbers of the function $g(x) = xe^{-2x}$, if any.

   Additionally, find the open intervals on which the function is increasing or decreasing and locate all relative extrema.

4. **(6 points)** Below is a graph of a function f. On the same viewing window, sketch a graph of its derivative f'.

5. Consider the function $f(x) = x^3 - 9x$.

    (a) **(2 points)** Find all of the critical numbers of f.

    (b) **(3 points)** Find the intervals on which f is increasing and decreasing.

    (c) **(2 points)** Locate any relative extrema.

    (d) **(3 points)** Find the intervals on which f is concave upward and concave downward.

    (e) **(2 points)** Find any points of inflection.

    (f) **(2 points)** Find any x-intercept(s).

    (g) **(3 points)** Make a complete, clear, neat sketch of the graph of f by using parts (b)-(f) from above.

Answer **TRUE** or **FALSE**. No work is necessary. Each problem is worth 3 points.

6. _____ The graph of a function may have both a horizontal asymptote and an oblique asymptote.

7. _____ If $f'(x)>0$ for $x<3$ and $f'(x)<0$ for $x>3$, then $f(3)$ is a local maximum.

8. _____ Let $p(x)$ be a polynomial function such that $p(-2)=5$, $p'(-2)=0$, and $p''(-2)=-9$. The point $(-2,5)$ is an inflection point of the graph of p.

9. _____ If $f'(x)<0$ for all x in (a,b), then f is decreasing on (a,b).

10. _____ If $y=f(x)$, f is increasing and differentiable, and $\Delta x>0$, then $\Delta y>dy$.

11. _____ If there is a minimum of $f(x)$ at $x=a$, then $f'(a)=0$.

12. _____ Rolle's Theorem is a special case of the Mean Value Theorem.

13. _____ Given the graph of $f'(x)$, you can uniquely construct the graph of $f(x)$.

14. _____ The maximum value of $y=6\cos x+2\sin x$ is 8.

BONUS (Optional 5 points)

Consider the function $f(x)=\dfrac{1}{5x^2+2}$. <u>Without</u> using calculus and <u>without</u> using the graph of f, explain why this function has a single maximum value.

DIRECTIONS: Calculators are permitted on this part of the exam. However, answers based solely on calculator results are unacceptable. You must still <u>show all work</u> to receive full credit. Good luck.

15. (**5 points**) Locate the absolute extrema of the function $y = 5x^{2/3} - x^{5/3}$ on the interval $[-1, 4]$.

16. (**6 points**) Find all points of inflection to the graph of $h(x) = 2x - \tan x$ and discuss the concavity of the graph of the function. Only consider the graph on the interval $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right)$.

17. **(9 points)** A piece of sheet metal is rectangular, 5 ft wide and 8 ft long. Congruent squares (same size) are to be cut from its four corners. The resulting piece of metal is to be folded and welded to form a box with an open top, as shown below. How should this be done to get a box of largest possible volume? **Be sure to show all work here.**
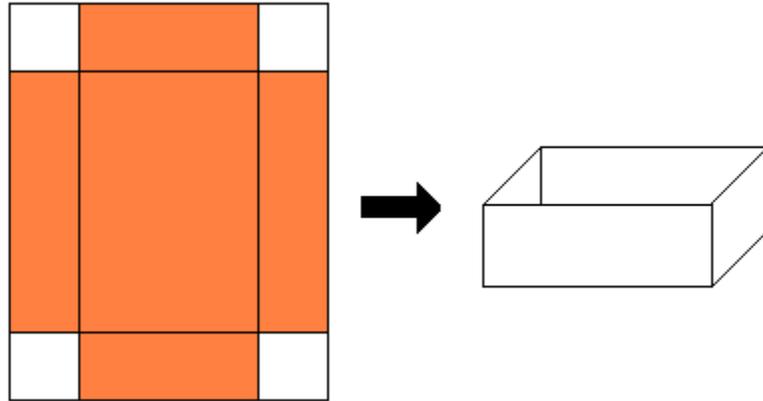
18. **(7 points)** Use differentials to approximate the value of $\sqrt{99.4}$. Specifically, make use of the formula $f(x+\Delta x) \approx dy + f(x)$. Compare the answer with that of a calculator.