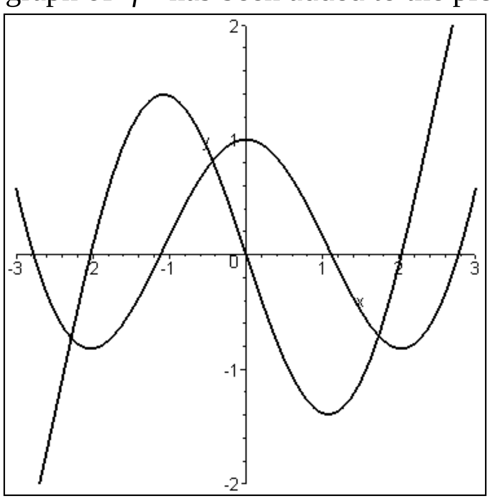
MTH 150
Sample Exam 3
Key

1. The function is continuous on $[-2,2]$ and differentiable on $(-2,2)$. Furthermore, $f(-2)=f(2)$. Therefore Rolle's Theorem can be applied. You should find three c values: $1,\ 0,\ -1$.

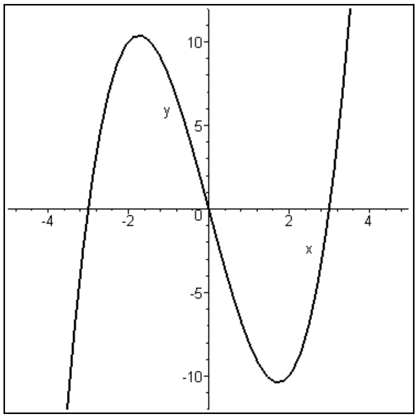
2. Use the formula $D=RT$ or $R=\frac{D}{T}$. Here, $R=\frac{30\ \text{miles}}{25\ \text{minutes}}=\frac{30\ \text{miles}}{\frac{25}{60}\ \text{hour}}=72\ \text{mph}$. Thus, the average speed over the thirty mile trip was 72 miles per hour. The Mean Value Theorem says that at some point during the trip, you were traveling <u>exactly</u> 72 mph.

3. The critical number is $x=\frac{1}{2}$. The function is increasing on $(-\infty,1/2)$ and decreasing on $(1/2,\infty)$. There is a maximum at $\left(\frac{1}{2},\frac{1}{2e}\right)$.

4. The graph of f' has been added to the picture:



5. (a) $\pm\sqrt{3}$
   (b) increasing on $\left(-\infty,-\sqrt{3}\right)$, $\left(\sqrt{3},\infty\right)$; decreasing on $\left(-\sqrt{3},\sqrt{3}\right)$.
   (c) maximum: $\left(-\sqrt{3},6\sqrt{3}\right)$; minimum: $\left(\sqrt{3},-6\sqrt{3}\right)$
   (d) concave upward on $(0,\infty)$, concave downward on $(-\infty,0)$
   (e) the origin
   (f) $(0,0)$, $(\pm 3,0)$
   (g) A sketch:



6. False
7. False
8. False
9. True
10. False
11. False
12. True
13. False
14. False

15. First find the critical numbers by differentiating and then simplifying:

$$\begin{aligned} y' &= \frac{10}{3}x^{-1/3} - \frac{5}{3}x^{2/3} \\ &= \frac{10}{3x^{1/3}} - \frac{5x}{3x^{1/3}} \\ &= \frac{10-5x}{3x^{1/3}} \\ &= \frac{5(2-x)}{3x^{1/3}} \end{aligned}$$

From above, the critical numbers are $x = 0$ and $x = 2$. Now make a table including a check at the endpoints:

x	y
0	0
2	4.76
-1	6
4	2.52

The minimum is $(0,0)$ and the maximum is $(-1,6)$.

16. The graph is concave upward on $\left(-\frac{\pi}{2},0\right)$ and concave downward on $\left(0,\frac{\pi}{2}\right)$. The inflection point is at the origin.

17. Cut out a square with sides of length *x*. Then the height is *x* and the dimensions of the base of the box are $5-2x$ and $8-2x$. Since the volume of a rectangular box is length times width times height, we have $V(x)=x(5-2x)(8-2x)$. We wish to <u>maximize</u> this function. Notice that for any critical numbers to make sense, *x* must be positive but also less than 2.5 feet. Otherwise, the side whose length is 5 ft disappears. In short, the domain of $V(x)$ is $0<x<2.5$. First, you can expand $V(x)$ to get $V(x)=40x-26x^2+4x^3$. This leads to $V'(x)=40-52x+12x^2$. Setting this to zero we get $0=40-52x+12x^2$ or, after factoring, $0=4(3x-10)(x-1)$. The critical numbers are $x=\frac{10}{3}$ (impossible—see earlier comment) and $x=1$. Look at the sign of $V'(x)$ on a number line to verify that $x=1$ is a maximum. Therefore, we choose $x=1$ while the other two sides are $5-2(1)=3$ and $8-2(1)=6$. Hence, the dimensions are $6'$ by $3'$ by $1'$ leading to a volume of $18\ \text{ft}^3$.

18. $\sqrt{99.4}\approx 9.97$. A calculator gives about 9.96995 . . .