MTH 150   Name: _____
Exam 3
April 10, 2007

DIRECTIONS: This is a <u>closed book</u>, <u>closed notes</u> exam. <u>No electronic devices</u> are allowed (this means calculators, computers, cell phones, pagers, etc.). Be neat and <u>show all work</u> to receive full credit. Correct answers without the supporting evidence to back it up (except where indicated) receive only partial credit. Good luck.

1. (**8 points**) State Rolle's Theorem. Draw and label a picture that represents the theorem graphically.

2. (**6 points**) Let $x = c$ be a critical number of f such that $f'(c) = 0$. Sketch the graph of a continuous function f such that $f''(c) > 0$.
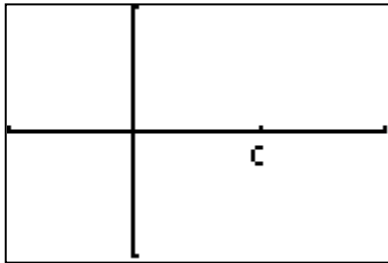
Problems 3-8 are multiple choice. Circle the letter of your choosing. No work is necessary.

3. (**5 points**) Use the Mean Value Theorem to complete the following statement. If a function f is continuous on $[2,6]$ and differentiable on $(2,6)$ with $f(2) = 5$ and $f(6) = -3$, then for some c in $(2,6)$, _____ .

   A. $f'(c) = 1$ B. $f(c) = 0$

   C. $f'(c) = 0$ D. $f(c) = 2$

   E. $f'(c) = -2$
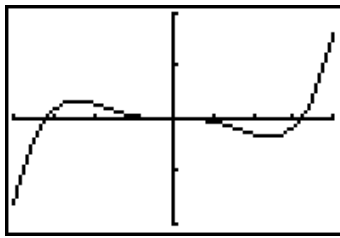
4. **(5 points)** Find the coordinates of the extreme value of $f(x) = (x-1)e^x$.

    A. $(0,0)$ B. $(-1, -2/e)$

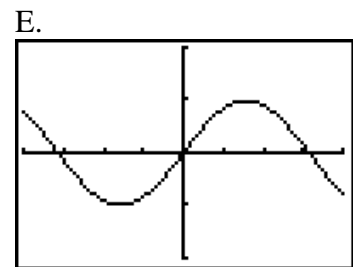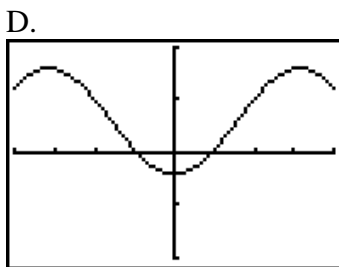   C. $(0,-1)$ D. $(1,0)$

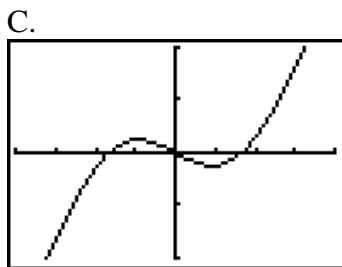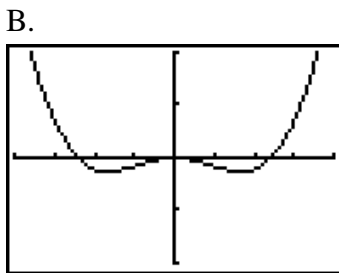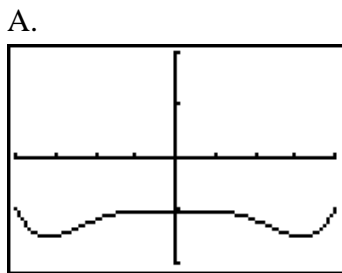   E. $(0,-e)$

5. **(5 points)** Consider the function $f(x) = \cos x - x$ on the interval $[0, 2\pi]$. Find the interval on which f is decreasing.

    A. $(0, 2\pi)$ B. $\left(0, \dfrac{3\pi}{2}\right)$

    C. $\left(\dfrac{3\pi}{2}, 2\pi\right)$ D. $\left(0, \dfrac{\pi}{2}\right)$

    E. $\left(\dfrac{\pi}{2}, 2\pi\right)$

6. **(5 points)** Consider the graph of f below:

    Which of the following could be the graph of f'?

    A.   B.

    C.   D.   E.

7. **(5 points)** Find the value of the absolute maximum of $f(x) = x^{2/3}$ on the interval $[-1, 8]$.

A. -1 B. 0

C. 1 D. 4

E. 8

8. **(5 points)** The Second Derivative Test is used to

A. determine intervals of increase/decrease   B. determine the concavity of the graph

C. find relative extrema   D. find points of inflection

E. None of these

DIRECTIONS: Calculators are permitted on this part of the exam. However, answers based solely on calculator results are unacceptable. You must still <u>show all work</u> to receive full credit. Good luck.

9. Consider the function $f(x) = x^5 - 5x$.

    (a) **(2 points)** Find all of the critical numbers of f.

    (b) **(3 points)** Find the intervals on which f is increasing and decreasing.

    (c) **(2 points)** Locate any relative extrema.

    (d) **(3 points)** Find the intervals on which f is concave upward and concave downward.

    (e) **(3 points)** Find any points of inflection.

    (f) **(3 points)** Approximate any *x*-intercept(s). Use the calculator.

    (g) **(4 points)** Make a complete, clear, neat sketch of the graph of f by using parts (b)-(f) from above.

10. (**9 points**) Find the critical numbers of the function $f(x) = (3-x)e^{x-3}$, if any. Additionally, find the open intervals on which the function is increasing or decreasing and locate all relative extrema.

11. (**9 points**) Find the intervals on which $h(x) = \dfrac{x}{x-1}$ is concave upward and concave downward and find any points of inflection, if applicable.

12. **(10 points)** A rectangular field is to have an area of $60,000 \text{ m}^2$. Fencing is required to enclose the field and to divide it in half. Find the dimensions of the field that will require the least amount of fencing. **Show all work here.**

13. **(8 points)** Compare the values of dy and Δy given that $y = \frac{1}{2}x^3$, $x = 2$, $\Delta x = dx = 0.1$. After computing these values, give an interpretation of what these values represent.