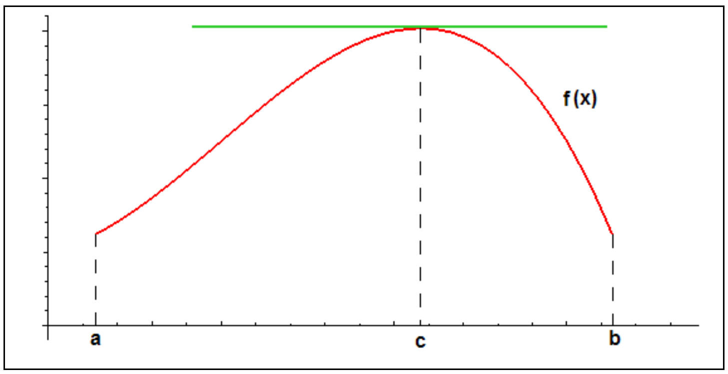
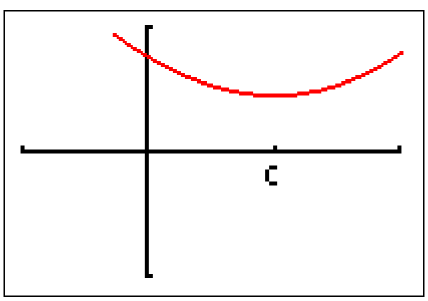
MTH 150
Exam 3
Solutions

Note: Some of these solutions contain many details; others do not.

1. Given a continuous and smooth function on the interval $[a,b]$ with $f(a)=f(b)$, there exists a number c in (a,b) where $f'(c)=0$ (i.e., a horizontal tangent). Here is a picture:



2. Something like the picture seen below (upward concavity):



3. E

4. C

5. Notice that $f'(x)=-\sin x-1\leq 0$ on any interval. Even though the graph "flattens out" when $x=\frac{3\pi}{2}$, the function continues to decrease. The best answer is the full interval $(0,2\pi)$ or A.

6. B

7. D

8. The Second Derivative Test, like the First Derivative Test, is used to locate extreme values. It uses concavity to answer this question but the results on concavity are local. The test doesn't give the full story on concavity. The best answer is C.

9. (a) $x = \pm 1$

(b) increasing on $(-\infty,-1)\cup(1,\infty)$; decreasing on $(-1,1)$

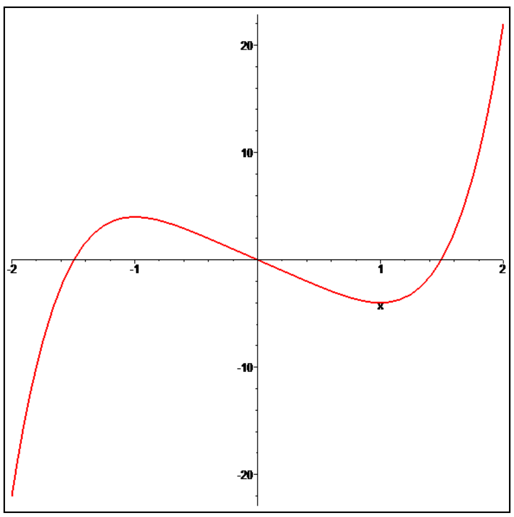
(c) $(-1,4)$ is a local max; $(1,-4)$ is a local min

(d) concave upward on $(0,\infty)$; concave downward on $(-\infty,0)$

(e) point of inflection at the origin

(f) approximately ± 1.4953 and the origin from above

(g)



10. Critical number at $x = 2$; increasing on $(-\infty,2)$ while decreasing on $(2,\infty)$; a maximum at $(2,e^{-1})$.

11. First get $h'(x) = \dfrac{-1}{(x-1)^2}$. Rewriting, $h'(x) = -1(x-1)^{-2}$. Another differentiation gives $h''(x) = -1(-2)(x-1)^{-3} = \dfrac{2}{(x-1)^3}$. A good number of you used the Quotient Rule to get $h''(x)$ (which is fine ☺) but you should notice that the method above is more efficient. It also results in a far less complicated expression; the Quotient Rule set-up requires more simplification. Study the sign of $h''(x)$ on either side of $x = 1$ for the concavity results.

12. 200 m × 300 m

13. Since $\frac{dy}{dx} = f'(x)$, we can write $dy = f'(x)dx$. From this, you should get $dy = \frac{3}{2}x^2 dx = \frac{3}{2}(2)^2(0.1) = 0.6$. This number gives the change in the tangent line from $x = 2$ to $x = 2.1$. It should be a fairly good approximation to the actual change in the function $y = \frac{1}{2}x^3$ from $x = 2$ to $x = 2.1$. To check this,

$$\begin{aligned}
\Delta y &= f(x+\Delta x) - f(x) \\
&= f(2+0.1) - f(2) \\
&= f(2.1) - f(2) \\
&= \frac{1}{2}(2.1)^3 - \frac{1}{2}(2)^3 \\
&= 0.6305
\end{aligned}$$