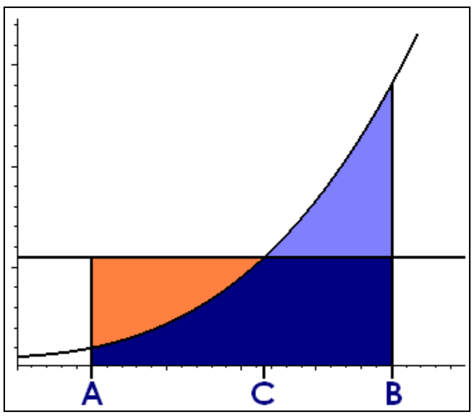
MTH 150
Sample Exam 4
Key

1. Answer: $\frac{\theta^5}{5}+\sec\theta+C$. Check that $\frac{d}{d\theta}\left(\frac{\theta^5}{5}+\sec\theta+C\right)=\theta^4+\sec\theta\tan\theta$.

2. For $f(x)$ continuous on $[a,b]$, there exists a value c in $[a,b]$ such that $\int_a^b f(x)\,dx=(b-a)f(c)$. In plain terms, there exists a rectangle with width $b-a$ whose area is exactly equal to the area given by $\int_a^b f(x)\,dx$. A picture:



The darker area shows where the area underneath the curve and the area of the rectangle *overlap*.

3. (a) $\pi-2$ (b) -1 (c) $\pi+\frac{13}{2}$

4. $F'(x)=\frac{\arccos\left(x^{3/2}\right)}{2\sqrt{x}}$

5. $\frac{2}{5}(1+x)^{5/2}-\frac{2}{3}(1+x)^{3/2}+C$

6. $\frac{1}{2}\arctan x^2+C$

7. $6\ln\left|\sec t+6\right|+C$

8. $-\frac{1}{6}\ln\left|\cos\left(3x^2\right)\right|+C$

9. (a) 14

(b) Let $u = x+2$. Then since $x = -2 \to 3$, we must have $u = 0 \to 5$. Thus,

$\int_{-2}^{3} f\left(x+2\right)dx = \int_{0}^{5} f\left(u\right)du = 4$, as given in the problem.

(c) 8

10. FTOC 1: $\int_{a}^{b} F'\left(x\right)dx = F\left(b\right) - F\left(a\right)$. The area underneath a curve can be found by first finding an antiderivative and then subtracting at the endpoints.

FTOC 2: $\frac{d}{dx}\int_{a}^{x} f\left(t\right)dt = f\left(x\right)$. This part, more or less, illustrates how differentiation and integration are inverse operations. Given any function, (indefinite) integration followed by differentiation will give back this function.

11. (a) If using the left endpoints, the heights of the rectangles will be calculated at 1, 1.25, 1.5, and 1.75. The width of each rectangle is 0.25. Thus the approximate area is $\frac{1}{4}\left[f\left(1\right)+f\left(1.25\right)+f\left(1.5\right)+f\left(1.75\right)\right] = 7.078125$.

(b) $\frac{25}{4}$

12. (a) 2 (b) 0