MTH 150
Exam 4
Solutions

Note: Some of these solutions show many details; others do not.

1. (a) 7.078125 (b) $25/4$

2. (a) -2

(b) First notice that $\int \frac{1}{1+x^2}\,dx = \arctan x + C$. From this, we get

$$\begin{aligned}\int_0^1 \frac{4}{1+x^2}\,dx &= 4\int_0^1 \frac{1}{1+x^2}\,dx \\ &= 4\arctan x\Big|_0^1 \\ &= 4\big(\arctan 1 - \arctan 0\big) \\ &= 4\left(\frac{\pi}{4} - 0\right) \\ &= \pi.\end{aligned}$$

3. A 4. B 5. A 6. C
7. E 8. B 9. A 10. C

11. Let $u = 6x$ so $du = 6dx$ or $\frac{du}{6} = dx$. Then

$$\begin{aligned}\int \sec(6x)\,dx &= \int \sec u \cdot \frac{du}{6} \\ &= \frac{1}{6}\int \sec u\ du \\ &= \frac{1}{6}\ln\left|\sec u + \tan u\right| + C \\ &= \frac{1}{6}\ln\left|\sec(6x) + \tan(6x)\right| + C.\end{aligned}$$

12. $\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

13. If you split the integral in two, each one fits a recognizable antidifferentiation rule:

$$\int \frac{x+2}{\sqrt{4-x^2}}\,dx = \int \frac{x}{\sqrt{4-x^2}}\,dx + \int \frac{2}{\sqrt{4-x^2}}\,dx$$

For the first integral, notice that substitution works (use $u = 4 - x^2$). Then

$$\begin{aligned}\int \frac{x}{\sqrt{4-x^2}}\,dx &= \int x\left(4-x^2\right)^{-1/2}\,dx \\ &= -\tfrac{1}{2}\int -2x\left(4-x^2\right)^{-1/2}\,dx \\ &= -\tfrac{1}{2}\frac{\left(4-x^2\right)^{1/2}}{1/2} \\ &= \boxed{-\sqrt{4-x^2}}\end{aligned}$$

The other integral fits the arcsine rule:

$$\begin{aligned}\int \frac{2}{\sqrt{4-x^2}}\,dx &= 2\int \frac{1}{\sqrt{4-x^2}}\,dx \\ &= 2\int \frac{1}{\sqrt{2^2-x^2}}\,dx \\ &= \boxed{2\arcsin\left(\tfrac{x}{2}\right)}\end{aligned}$$

Then the answer is found by adding the two boxed answers:

$$\boxed{-\sqrt{4-x^2} + 2\arcsin\left(\tfrac{x}{2}\right) + C}$$

14. $\tfrac{1}{2}x^2 - 4x + 6\ln\left|x+1\right| + C$