

CUTTING & STACKING PAPER

Take a sheet of paper, cut it in half (maybe fold it first), and then place one half on top of the other half to create a pile of paper with a height equal to the thickness of two sheets of paper. Take that pile, cut it in half (maybe fold it first), and place one half on top of the other. The resulting pile would have a height equal to four sheets of paper. Continue this process...

Question: How high would the pile be if you repeated this 50 times?

Source: [Trinter & Garofalo, 2011](#)

It's nice to start with a table:

# folds	thickness
0	0.004
1	0.008
2	0.016
3	0.032 = 8 (0.004)
4	0.064
5	0.128
6	0.256
7	0.512
8	1.024

The key here is trying to figure out how the number of folds (F) determines the thickness (T) of the paper. Looking at line 4 of the above board, we ask, "How can you obtain $T = 8(0.004)$ from $F = 3$?" Notice that $8 = 2 \times 2 \times 2 = 2^3$. Therefore, we see $T = 2^3(0.004)$. This leads us to the formula $T = 2^F(0.004)$. (Check that this works for other values of F .) Now the original question can be answered by letting $F = 50$. Here are some boards of work:

~~1,125,899,906,842,624~~
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 $\times 0.004$

 $4,503,599,627,370.496$
 $t = .004(2)^n$

$n = \text{Paper cents}$
 $t = \text{thickness}$

# of folds	layers of Paper
1	2)x2
2	4)x2
3	8)x2
4	16)x2
5	32)x2
6	64)x2

$2^{(F)} = P$
 $(2^{(F)})0.004 = T$
 $2^{60} = 1.1258999 \times 10^{18}$
 $\times 0.004 = 7$
 4.5035996×10^{12}

$F = \# \text{ of folds}$
 $P = \text{layers of Paper}$
 $T = \text{total thickness}$

Folds	Layers	Thickness
0	1	.004
1	2	.008
2	4	.016
3	8	.032
4	14	.064

$2^F = L$
 $L \cdot 0.004 = T$
 $T = 2^F(0.004)$
 $T = 4 \text{ trillion in}$
 $\frac{T}{12} = 375 \text{ billion ft}$
 $\frac{T/12}{5280} = T \text{ in Miles}$

$71,079,539.573397$
 Miles

Note: The board directly above should have the number 16 (not 14) in the “layers” column. As we discussed in class, the value (in inches) is **so large** that it makes sense to convert it to feet and then miles to interpret its meaning:

$2^F \leftarrow \# \text{ folds}$

$4.5 \times 10^{12} \cancel{\text{ in}} \times \frac{1 \cancel{\text{ ft}}}{12 \cancel{\text{ in}}} \times \frac{1 \text{ mi}}{5280 \cancel{\text{ ft}}}$

$\approx 71,000,000 \text{ miles!}$