MATH 166
Initial Skills Packet--Answers

ARITHMETIC/NUMERICAL REASONING

1. 11/30

2. It simplifies to 400. Large.

3. Given any exponential equation, a logarithm provides a way of isolating the exponent. For example, suppose we wished to solve the equation $3^x = 81$. If we ponder this for a moment, we see that $x = 4$. Rather than saying, "4 is the power to which we need to raise 3 in order to get 81" we say $4 = \log_3 81$.

4. You could offer a few logical reasons. Here are two. (a) "Dividing by" means we are taking a number and subdividing it into equal parts. For example, $\frac{10}{2} = 5$ because if we wish to divide 10 items into two piles, each pile will contain 5 items. Using this paradigm, $\frac{10}{0}$ implies we are trying to subdivide 10 items into zero piles. How many items would each pile contain? (b) Another approach is to examine what happens when we attempt to divide a fixed number by values that *approach* zero. For example, $\frac{4}{1} = 4,\ \frac{4}{0.1} = 40,\ \frac{4}{0.01} = 400,\ \frac{4}{0.001} = 4000$, etc. As we continue to subdivide 4 items, the result "moves" toward larger and larger values, but no well-defined value in particular.

5. (a) $-1 < \cos\left(\frac{7\pi}{10}\right) < 0$ (b) $2 < \log_4 30 < 3$ (c) $0 < e^{-1} < 1$

6. Both are close to 3 ($\pi \approx 3.14$ while $e \approx 2.72$). The constant π should be familiar from geometry (e.g., area/circumference/volume of various figures). The constant e appears in many contexts—finance, nature, and gaming, just to name three.

7. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$ so this is a positive (but small) number. Remember that negative exponents do not imply negative numbers.

ELEMENTARY ALGEBRA

8. $\frac{2x^2 + 3x + 3}{(x+1)(x+3)}$ 9. $2 \pm \frac{\sqrt{14}}{2}$ 10. $y = -\frac{3}{5}x + \frac{13}{5}$

11. $-\frac{2}{x^5}$

12. (a) $(3x-1)(x-1)$
(b) prime
(c) prime
(d) $(x-5)(x+5)$
(e) prime
(f) $(3x^2-2)(x+1)(x-1)$
(g) $(2y-3)(4y^2+6y+9)$

ADVANCED ALGEBRA & FUNCTIONS

13. (a) -13
(b) 6
(c) $-2x^2-4hx-2h^2+5x+5h-6$
(d) $a-b-4$
(e) $-2x^2+21x-58$
(f) $-2x^2+5x-10$

14. $\frac{1}{\sqrt{x+h}+\sqrt{x}}$

15. $-\frac{1}{x(x+h)}$

16. Dividing by $a-b$ is the mathematical error. Notice $a=b$ so this means $a-b=0$. If division by zero were permitted, we could make a host of ridiculous claims, including $2=1$. See item #4 for more on division by zero.

17. Recall that $f^{-1}(x)$ denotes the inverse (function) of f. Thus, $f(5)=-2$ implies $f^{-1}(-2)=5$. Since the function f maps 5 to -2, the inverse function f^{-1} does the opposite; it maps -2 to 5.

18. (a) Domain: $(-\infty,\infty)$; Range: $[7,\infty)$; Inverse: $f^{-1}(x)=4+\sqrt{x-7}$

(b) Domain: $[5/2,\infty)$; Range: $[0,\infty)$; Inverse: $g^{-1}(x)=\frac{1}{2}(x^2+5)$, $x\geq 0$

TRIGONOMETRY

19. (a) $\sqrt{2}/2$
(b) $1/\sqrt{3}$
(c) $1/2$

20. (a) $\pi/6$
(b) $\pi/3$
(c) undefined

21. The sine and cosine functions are odd and even functions, respectively. For example, an even function is one that, when graphed, is symmetric with respect to the *y*-axis (think of something like $y=x^2$). The cosine function has exactly this property.

22. (a) Domain: $(-\infty,\infty)$; Range: $[-1,1]$; Inverse: $k^{-1}(x)=\frac{1}{2}\cos^{-1}x$

(b) Domain: All real numbers except $x = \frac{n\pi}{2}$, $n = \pm 1, , \pm 3, \pm 5 \ldots$; Range: $(-\infty, \infty)$;

Inverse: $j^{-1}(x) = \tan^{-1} x$

MISCELLANEOUS

23. (a) False
(b) True
(c) False
(d) True
(e) True
(f) True
(g) False
(h) False
(i) True
(j) False
(k) True
(l) True
(m) True
(n) False
(o) False